Load forecasting for economic power system operation

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LOAD FORECASTING FOR ECONOMIC POWER SYSTEM OPERATION

BY

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ABSTRACT

Short-term load forecasting is important for reliable and economic operation of a power system. The aim of this research is the development of statistical models capable of predicting the short-term total system load for a small, isolated power system, utilising both historical demand patterns and the underlying relationship between electrical demand and meteorological conditions.

An essential prerequisite for the implementation of such a model is the continuous monitoring of meteorological conditions. To enable such monitoring a data acquisition system has been developed and installed at a suitable location.

Initially, the value of forecasting was ascertained by considering the economic implications for system operation in the absence of forecasting. Subsequently, the Box-Jenkins time series approach was applied to the short-term load forecasting problem. Both univariate and multivariate (with weather inputs) models have been developed with a reasonable degree of success.

An off-peak load installation has also been monitored over an extended period and results from this experiment support the use of off-peak load for emergency reserve substitution. The practical aspects of using this low priority load for distributed load shedding have also been examined.

Although the time series techniques applied in this study are well proven, the main contribution of this thesis has been the application of these statistical methods to the actual, local power system. Results indicate that by employing the multivariate technique an increase in accuracy over the univariate method was attainable. However, the improvement was not consistent.

Another departure was the use of the forecasts produced to investigate the effect of forecast accuracy on system operational costs. The univariate technique provided sufficient accuracy (especially over 2h ahead) to approach optimum operation when used with an economic loading algorithm, assumming that an AGC system would make the necessary on-line adjustments to synchronised generation to account for forecasting errors.

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Statistical Analysis

ACF	Autocorrelation function
ACVF	Autocovariance function
AIC	Akaike information criterion
AR	Autoregressive
ARMA	Mixed autoregressive moving average
CCF	Cross-correlation function
Cov	Covariance
e(t)	White noise
E[Y(t)]	Expected value of Y(t)
FPE	Final prediction error
MA	Moving average
MAPE	Mean absolute percentage error
b	Pure time delay between input and output
p	Autogressive order
q	Moving average order
r	Denominator order in multivariate model
s	Numerator order in multivariate model
tc	Trend constant
$x_i(t)$	Input variable i at time t
y(t)	Actual value of output variable
$\hat{y}(t)$	Estimate of output variable

α	Transformed and differenced input vector	
β	Transformed and differenced output series	
δ	Denominator factor on input series in multivariate model	
ω	Numerator factor on input series in multivariate model	
B	Backward shift operator, where y(B)=y(t)-y(t-1)	
σ	Standard deviation	
γ	Autocovariance function	
ρ	Autocorrelation value	
$N(\mu,\sigma^2)$	Normally distributed random variable, mean=μ, variance=σ^2	
∇	Difference operator, such that $\nabla y(t)=y(t)-y(t-1)$	
χ^2	Chi-squared random variable	
ϕ_i	AR coefficient in univariate model	
θ_i	MA coefficient in univariate model	
ϕ_{si}	Seasonal AR coefficient in univariate model	
θ_{si}	Seasonal MA coefficient in univariate model	
υ_i	Impulse response weights	

Economic Loading Algorithm

N	Number of generating units	
P	Number of scheduling intervals	
D_j	Demand at scheduling point j	(MW)
T_j	Time interval j	(h)
a_i	No-load cost of unit i	(£)

b_i	Incremental cost of unit i	(£/MWh)
xl_i	Lower limit of unit i	(MW)
xu_i	Upper limit of unit i	(MW)
x^j_i	Unit i output during T_j	(MW)
λ^j_i	1 if unit i 'on' during T_j, otherwise 0	
ϕ^j_i	1 if unit i starts during T_j, otherwise 0	
S^j_i	Start-up cost of unit i at the beginning of interval T_j	(£)
R	System reserve coverage	
Z	System generation cost	(£)
S_{ni}	nth slope of the reserve characteristic of unit i	
C_{ni}	nth intercept of the reserve characteristic of unit i	

1.1 General Introduction

Forecasting is one of the most pervasive and important elements of managerial decision-making because the ultimate effect of a decision generally depends on the outcome of factors that cannot be seen at the time the decision is being made. The role of forecasting cuts across all fields of management - finance, marketing, production, engineering, operations research, business economics, and public administration. This work is primarily concerned with the contribution that load forecasting can make to power system operation and energy management.

Load forecasting has always been an integral part of electric power system operations. The necessity for estimating the power system load expected at some time in the future is apparent when it is remembered that generating plant capacity must be available to balance exactly any network load at whatever time it occurs. Many techniques have been investigated to solve the problem of load forecasting in the last 15 to 20 years. These techniques have been classified, according to the area of application, into long-, medium-, and short-term load forecasting.

Long-term load forecasts of 5 to 20 years ahead are needed for scheduling construction of new generating capacity as well as determination of prices and regulatory policy. Medium-term load forecasts of a few months to 5 years ahead are needed for maintenance scheduling, coordination of power sharing arrangements and setting of prices, so that demand can be met with available capacity.

In the short-term the variation of the system load must be known in order that prior warning of output requirements may be given to power stations, enabling limitations on the boiler fuel feed rates, and generator rate of change of output constraints, to be observed. Furthermore, the economic schedule for the start-up and shut-down of plant is dependent on an estimate of the system load so that the cost of providing spinning reserve for system security can be minimised. So, the principal objective of the load forecasting function is to provide load predictions for:

(a) Basic generation scheduling functions

The primary application of the short-term load forecasting function is to drive the scheduling functions that determine the most economic commitment of generation sources consistent with reliability requirements, operational constraints and policies, and physical, environmental and equipment limitations. For purely thermal systems, the load forecasts are needed by the unit commitment function to determine the minimal cost hourly strategies for the start-up and shut-down of units to supply the forecast load.

(b) Power System Security Assessment

A second application of short-term load forecasting is for predictive assessment of the power system security. The system load forecast is an essential data requirement of the off-line network analysis function for the detection of future conditions under which the power system may be vulnerable. This information permits dispatchers to prepare the necessary corrective actions (bringing peaking units on line, load shedding, power purchases, switching operations) to operate the power system securely.

(c) Timely dispatcher information

The third application of short-term load forecasting is to provide dispatchers with timely information, i.e. the most recent load forecast, with the latest weather prediction and random behaviour taken into account. The dispatchers need this information to operate the power system economically and reliably.

The term 'short' is used to imply prediction times of the order of hours, the basic quantity of interest being the hourly/half-hourly total system load. Such short-term load forecasting within the Northern Ireland Electricity (NIE) system, which is a small to medium-sized isolated power system with an installed capacity of just over 2000 MW, is the focus of this research. However, before continuing, a logical step is to consider the need or justification for short-term load forecasting in relation to the NIE system. Thus, the following sections provide an outline of the NIE system and an economic justification for forecasting.

1.2 NIE System

NIE provides Northern Ireland, an area of 5,462 square miles, with all its public electricity requirements. Through its 5,989 staff it serves a community of one and a half million people including approximately 527,000 domestic consumers and 77,770 commercial, industrial and agricultural users. The split between domestic and commercial/industrial demand is 44%/56% respectively. In 1990-91 NIE achieved a turnover of £404 million and the average growth in sales over the last five years has been 2.8% per annum.

To meet a maximum demand of 1447 MW, four power stations supply electricity through a transmission and distribution network of over 50,000 kilometres. These four generation stations have a total of 24 generating units which are controlled from Castlereagh

House Control Centre in Belfast, Fig 1.1. The units comprise coal-fired, oil-fired, and gas-turbine generators. Table 1.1 shows the composition of NIE's generating capacity. A typical morning peak of 900 MW requires about ten generating units to be synchronised.

NIE is currently undergoing a transition from the public to the private sector. Privatisation has also led to an increased interest by gas suppliers in bringing gas to Northern Ireland for power generation and to a renewed prospect of interconnection with Scotland and the Republic of Ireland.

Power Station	Number of Units	Capacity
Kilroot	2×250 MW oil mode	500 MW
	2×180 MW coal mode	or 360 MW
	1×60 MW gas-turbine	60 MW
Ballylumford	3×200 MW, 3×120 MW oil-fired	960 MW
	2×60 MW gas-turbine	120 MW
Coolkeeragh	5×60 MW, 2×30 MW oil-fired	360 MW
	2×30 MW gas-turbine	60 MW
Belfast West	3×60 MW, 2×30 MW coal-fired	300 MW
	Total Kilroot on oil	2360 MW
	Total Kilroot on coal	2220 MW

Table 1.1 NIE System Capacity

1.3 Economic Justification

Many techniques have been investigated to solve the problem of short-term load forecasting, but with limited emphasis on evaluating the benefits of forecasts. The following analysis aims to assess the timeliness and accuracy of short-term load forecasts in relation to power system operations, especially economic scheduling, and hence provide a justification for this research. The economic loading algorithm employed here has been developed by fellow researchers and is described in detail elsewhere (1,2). An outline of the economic loading problem will be given first.

Figure 1.1 NIE System

1.3.1 Load Prediction and Generation Capacity

The basis of system operation is the balance between electrical generation and the load demand as and when it occurs. A thermal unit needs several hours of preparation in readiness for synchronisation, while a hot unit can be synchronised within fifteen minutes and fully loaded in 30 to 60 minutes. Hence decisions on plant commitment must be made hours ahead of the event on the basis of the predicted demand. Thus system operation engineers need accurate load forecasting both for periods greater than unit lead times to operate the scheduling function and at smaller fixed intervals for economic dispatch. So the economic loading of generators consists of an optimal generator schedule and the subsequent dispatch of the committed generating units at least cost.

Scheduling of Generating Plant

Having estimated the total capacity required, by summing expected demand and reserve capacity, and knowing the capacity that is available, a decision has to be made as to which generators should be 'on' (committed) to satisfy the demand. This operation is termed unit commitment - the distribution of the total generation requirement of the system among the generation units over the total period considered for optimum system economy, within the limits permitted by system operation constraints and requirements.

So, having N generating units available, and a forecast of the demand to be served, the unit commitment problem is the determination of which subset of N can be employed to satisfy the demand at minimum operating cost.

Unit commitment, which is a dynamic scheduling problem, is crucial for economic system operation for the following reasons:

(a) Most generating units have start-up costs which are determined by how long the units have been off-line.

(b) Most units have minimum up and down times and rate limit constraints which have to be observed.

(c) The process of shut-down and start-up needs to be implemented in an economical manner.

Thus these constraints on a unit have to be observed while ensuring that at any time there is sufficient on-line capacity to:

(i) meet the expected load plus transmission and distribution losses,

(ii) satisfy the system reserve requirements.

This dynamic scheduling problem is complex, due to the fact that if we have N generating units to be scheduled over P periods, there are 2^{PN} combinations to be considered. A dynamic programming approach was used to solve this problem.

Economic Dispatch

Generation dispatch is a term used to describe the calculation and transmission to power stations of the generation required in the event. The aim of dispatch is to achieve minimum generation cost subject to satisfactory quality of supply and satisfactory plant loading and voltage under normal outage conditions.

Economic dispatch basically determines the optimum loading for that subset of the N available units which have been committed. Thus the economic dispatch algorithm is an essential sub-algorithm of the generation scheduling algorithm.

The most familiar dispatch methods are the merit order, heuristic programming and linear programming approaches. The linear programming method has been applied here.

The economic loading process employed here consists of two parts : dynamic programming and economic dispatch. Dynamic programming is a methodical procedure which systematically evaluates a large number of possible decisions in a multi-step problem. The dynamic programming algorithm used here works forward in time, and stores a single backward path to the first scheduling interval, the retained paths being the ones with the lowest accumulated costs. The commitment process uses a constraint directed approach (1) at each scheduling interval. Economic dispatch finds the optimum unit outputs required to meet a demand subject to the various constraints. The problem is formulated as a linear program and solved using the dual simplex method. This economic loading process is outlined in Appendix 1.

1.3.2 Forecasting Credit Evaluation

To perform a forecasting credit evaluation for the NIE system, 24-h economic loading calculations were carried out for both 'ideal' prediction and 'no-look ahead' operation, the difference in cost, providing an indication of the maximum possible forecasting credit.

Economic Loading Calculations

Thirteen units were employed, Table 1.2, with four 'must-run' units (1,4,9,10) being used (system loading and reserve data are detailed in Appendix 1). Rate limits were not activated and reserve cover was R=0.68, indicating that a proportion R of each infeed must be matched by spinning reserve from other units to limit possible load shedding. A limit was placed on maximum unit output by specifying an impact factor of 0.22, i.e, maximum unit output cannot exceed 0.22 $\times$ demand. All units were assummed to be initially on prior to each loading calculation. Half-hourly averaged demand data for Thursday 21st December 1989 was considered, Table 1.3, Fig. 1.2.

a. 'Ideal' Prediction

A normal 48-point loading calculation was performed, Fig. 1.3, assuming that the demand variation over the 24-hour period under consideration was ideally forecasted.

Fuel Costs = £235,279

Interval	Winter Profile	Summer Profile
1	934	525
2	886	512
3	851	508
4	828	486
5	836	468
6	801	451
7	760	437
8	737	436
9	717	427
10	702	414
11	690	417
12	698	432
13	721	475
14	765	546
15	818	659
16	945	745
17	1057	822
18	1078	850
19	1104	866
20	1099	860
21	1089	844
22	1086	834
23	1073	844
24	1091	848
25	1100	854
26	1093	833
27	1069	793
28	1037	777
29	1031	769
30	1022	763
31	1030	768
32	1070	784

Interval	Winter Profile	Summer Profile
33	1151	821
34	1217	874
35	1231	879
36	1212	856
37	1156	805
38	1112	748
39	1084	709
40	1053	680
41	1036	661
42	1016	657
43	974	648
44	956	651
45	948	677
46	924	681
47	917	634
48	934	531

Table 1.3 Demand Profiles

Unit	Power Station	Mode	
1	Kilroot	Oil	250 MW
4		Coal	180 MW
5	Ballylumford	Oil	120 MW
6		Oil	120 MW
7		Oil	120 MW
8		Oil	200 MW
9		Oil	200 MW
10		Oil	200 MW
11	Gas Turbine		60 MW
12	Gas Turbine		60 MW
13	Coolkeeragh	Oil	60 MW
15		Oil	60 MW
16		Oil	60 MW

Table 1.2 Units Employed in Loading Calculations

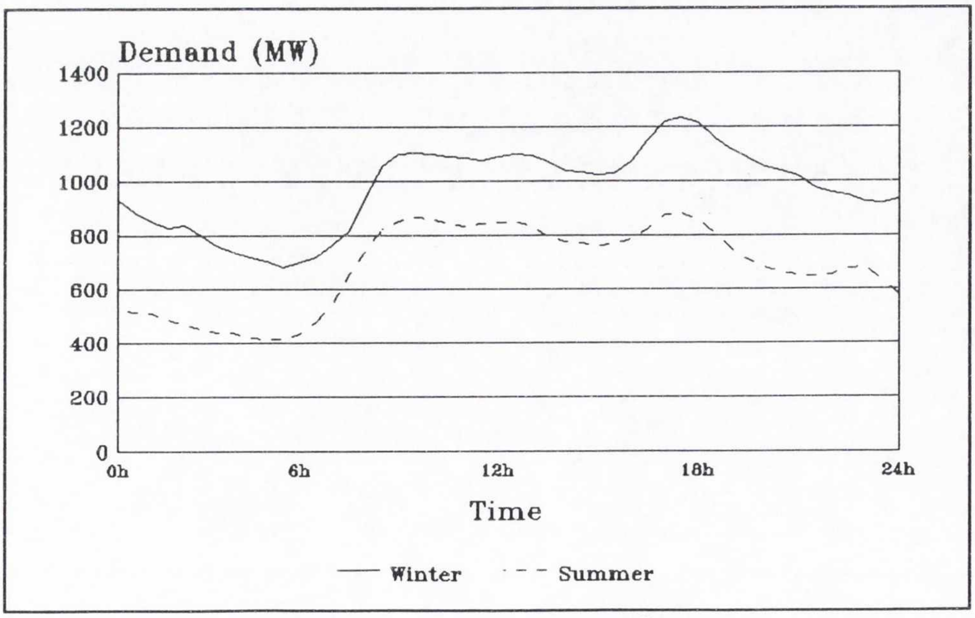

Figure 1.2 Winter and Summer Demand Profiles

b. 'No look-ahead' Operation

With 'no look-ahead' prediction the demand occurring at each half-hour is assumed to continue infinitely. Thus the start-up costs of any units coming on-line at any half-hour will be negligible over this infinite time period. Subsequently, the basic 'no look ahead' approach can be determined by suppressing start-up costs and performing the previous loading calculation, Fig. 1.4.

Fuel Costs = £232,808

However, the previous loading calculation does not consider the required lead times of units which come on-line and because, there is no forecast, the future need for such units cannot be foreseen. In the event they cannot be run-up immediately - unit lead times must be included. Following discussion with NIE System Operations staff, these were taken to be:

200 - MW unit .... 2h lead time

120 - MW unit .... 1.5h lead time

60 - MW unit .... 0h lead time

```
Run time     0 h 11 min 18 s

Reserve cover = 0.68

Fuel cost (£) =      235279.1
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Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
935.	0.5	163	0	0	180	50	50	0	169	163	157	0	0	0	0	0	0
910.	1.0	140	0	0	180	93	0	0	169	168	157	0	0	0	0	0	0
869.	1.5	141	0	0	180	50	0	0	169	168	157	0	0	0	0	0	0
840.	2.0	145	0	0	180	50	0	0	169	136	157	0	0	0	0	0	0
832.	2.5	146	0	0	180	50	0	0	169	127	157	0	0	0	0	0	0
819.	3.0	148	0	0	180	50	0	0	169	112	157	0	0	0	0	0	0
781.	3.5	131	0	0	169	0	0	0	169	161	148	0	0	0	0	0	0
749.	4.0	141	0	0	164	0	0	0	164	120	157	0	0	0	0	0	0
727.	4.5	148	0	0	159	0	0	0	159	100	157	0	0	0	0	0	0
710.	5.0	153	0	0	156	0	0	0	156	87	156	0	0	0	0	0	0
696.	5.5	153	0	0	153	0	0	0	153	97	138	0	0	0	0	0	0
694.	6.0	152	0	0	152	0	0	0	152	104	131	0	0	0	0	0	0
710.	6.5	153	0	0	156	0	0	0	156	87	156	0	0	0	0	0	0
743.	7.0	142	0	0	163	0	0	0	163	115	157	0	0	0	0	0	0
792.	7.5	137	0	0	162	0	0	0	169	167	154	0	0	0	0	0	0
882.	8.0	141	0	0	180	64	0	0	169	168	157	0	0	0	0	0	0
1001.	8.5	161	0	0	180	62	100	0	169	168	157	0	0	0	0	0	0
1078.	9.0	179	0	0	180	50	67	104	169	168	157	0	0	0	0	0	0
1101.	9.5	178	0	0	180	50	91	104	169	168	157	0	0	0	0	0	0
1092.	10.0	178	0	0	180	50	81	104	169	168	157	0	0	0	0	0	0
1084.	10.5	178	0	0	180	50	74	104	169	168	157	0	0	0	0	0	0
1088.	11.0	178	0	0	180	50	77	104	169	168	157	0	0	0	0	0	0
1080.	11.5	179	0	0	180	50	69	104	169	168	157	0	0	0	0	0	0
1082.	12.0	178	0	0	180	50	72	104	169	168	157	0	0	0	0	0	0
1096.	12.5	178	0	0	180	50	85	104	169	168	157	0	0	0	0	0	0
1097.	13.0	178	0	0	180	50	86	104	169	168	157	0	0	0	0	0	0
1081.	13.5	179	0	0	180	50	71	104	169	168	157	0	0	0	0	0	0
1053.	14.0	179	0	0	180	50	50	97	169	168	157	0	0	0	0	0	0
1034.	14.5	179	0	0	180	50	50	78	169	168	157	0	0	0	0	0	0
1027.	15.0	179	0	0	180	50	50	70	169	168	157	0	0	0	0	0	0
1026.	15.5	180	0	0	180	50	50	69	169	168	157	0	0	0	0	0	0
1050.	16.0	179	0	0	180	50	50	94	169	168	157	0	0	0	0	0	0
1111.	16.5	178	0	0	180	50	100	104	169	168	157	0	0	0	0	0	0
1184.	17.0	199	0	0	180	88	100	104	169	168	157	0	0	15	0	0	0
1224.	17.5	198	0	0	180	102	100	104	169	168	157	0	0	41	0	0	0
1222.	18.0	198	0	0	180	102	100	104	169	168	157	0	0	39	0	0	0
1184.	18.5	199	0	0	180	88	100	104	169	168	157	0	0	15	0	0	0
1134.	19.0	177	0	0	180	74	100	104	169	168	157	0	0	0	0	0	0
1096.	19.5	178	0	0	180	50	86	104	169	168	157	0	0	0	0	0	0
1069.	20.0	179	0	0	180	50	58	104	169	168	157	0	0	0	0	0	0
1045.	20.5	179	0	0	180	50	50	88	169	168	157	0	0	0	0	0	0
1026.	21.0	161	0	0	180	88	100	0	169	168	157	0	0	0	0	0	0
995.	21.5	161	0	0	180	56	100	0	169	168	157	0	0	0	0	0	0
965.	22.0	162	0	0	180	50	76	0	169	168	157	0	0	0	0	0	0
952.	22.5	162	0	0	180	50	63	0	169	168	157	0	0	0	0	0	0
936.	23.0	163	0	0	180	50	50	0	169	164	157	0	0	0	0	0	0
921.	23.5	165	0	0	180	50	50	0	169	147	157	0	0	0	0	0	0
926.	24.0	164	0	0	180	50	50	0	169	152	157	0	0	0	0	0	0

Figure 1.3 'Ideal' Prediction

Run time   0 h 10 min 25 s

Reserve cover = 0.68

Fuel cost (£) =   232808.3

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
935.	0.5	149	0	0	180	72	100	104	0	168	157	0	0	0	0	0	0
910.	1.0	140	0	0	180	93	0	0	169	168	157	0	0	0	0	0	0
869.	1.5	141	0	0	180	50	0	0	169	168	157	0	0	0	0	0	0
840.	2.0	138	0	0	175	102	100	0	0	168	155	0	0	0	0	0	0
832.	2.5	134	0	0	179	102	100	0	0	163	151	0	0	0	0	0	0
819.	3.0	134	0	0	180	88	100	0	0	163	151	0	0	0	0	0	0
781.	3.5	131	0	0	169	0	0	0	169	161	148	0	0	0	0	0	0
749.	4.0	141	0	0	164	0	0	0	164	120	157	0	0	0	0	0	0
727.	4.5	148	0	0	159	0	0	0	159	100	157	0	0	0	0	0	0
710.	5.0	153	0	0	156	0	0	0	156	87	156	0	0	0	0	0	0
696.	5.5	138	0	0	153	97	0	0	0	153	153	0	0	0	0	0	0
694.	6.0	138	0	0	152	97	0	0	0	152	152	0	0	0	0	0	0
710.	6.5	153	0	0	156	0	0	0	156	87	156	0	0	0	0	0	0
743.	7.0	142	0	0	163	0	0	0	163	115	157	0	0	0	0	0	0
792.	7.5	137	0	0	162	0	0	0	169	167	154	0	0	0	0	0	0
882.	8.0	141	0	0	180	64	0	0	169	168	157	0	0	0	0	0	0
1001.	8.5	161	0	0	180	62	100	0	169	168	157	0	0	0	0	0	0
1078.	9.0	179	0	0	180	50	67	104	169	168	157	0	0	0	0	0	0
1101.	9.5	178	0	0	180	50	91	104	169	168	157	0	0	0	0	0	0
1092.	10.0	178	0	0	180	50	81	104	169	168	157	0	0	0	0	0	0
1084.	10.5	178	0	0	180	50	74	104	169	168	157	0	0	0	0	0	0
1088.	11.0	178	0	0	180	50	77	104	169	168	157	0	0	0	0	0	0
1080.	11.5	179	0	0	180	50	69	104	169	168	157	0	0	0	0	0	0
1082.	12.0	178	0	0	180	50	72	104	169	168	157	0	0	0	0	0	0
1096.	12.5	178	0	0	180	50	85	104	169	168	157	0	0	0	0	0	0
1097.	13.0	178	0	0	180	50	86	104	169	168	157	0	0	0	0	0	0
1081.	13.5	179	0	0	180	50	71	104	169	168	157	0	0	0	0	0	0
1053.	14.0	179	0	0	180	50	50	97	169	168	157	0	0	0	0	0	0
1034.	14.5	160	0	0	180	96	100	0	169	168	157	0	0	0	0	0	0
1027.	15.0	161	0	0	180	88	100	0	169	168	157	0	0	0	0	0	0
1026.	15.5	161	0	0	180	88	100	0	169	168	157	0	0	0	0	0	0
1050.	16.0	179	0	0	180	50	50	94	169	168	157	0	0	0	0	0	0
1111.	16.5	178	0	0	180	50	100	104	169	168	157	0	0	0	0	0	0
1184.	17.0	199	0	0	180	88	100	104	169	168	157	0	0	15	0	0	0
1224.	17.5	198	0	0	180	102	100	104	169	168	157	0	0	41	0	0	0
1222.	18.0	198	0	0	180	102	100	104	169	168	157	0	0	39	0	0	0
1184.	18.5	199	0	0	180	88	100	104	169	168	157	0	0	15	0	0	0
1134.	19.0	177	0	0	180	74	100	104	169	168	157	0	0	0	0	0	0
1096.	19.5	178	0	0	180	50	86	104	169	168	157	0	0	0	0	0	0
1069.	20.0	179	0	0	180	50	58	104	169	168	157	0	0	0	0	0	0
1045.	20.5	179	0	0	180	50	50	88	169	168	157	0	0	0	0	0	0
1026.	21.0	161	0	0	180	88	100	0	169	168	157	0	0	0	0	0	0
995.	21.5	161	0	0	180	56	100	0	169	168	157	0	0	0	0	0	0
965.	22.0	162	0	0	180	50	76	0	169	168	157	0	0	0	0	0	0
952.	22.5	149	0	0	180	90	100	104	0	168	157	0	0	0	0	0	0
936.	23.0	149	0	0	180	74	100	104	0	168	157	0	0	0	0	0	0
921.	23.5	141	0	0	166	102	0	0	184	168	157	0	0	0	0	0	0
926.	24.0	150	0	0	180	63	100	104	0	168	157	0	0	0	0	0	0

Figure 1.4 'No-look-ahead' Operation

c. 'No look-ahead' Prediction with Lead Times Considered

The previous 'no look-ahead' loading calculation was repeated with unit lead times taken into account by controlling unit availability. The calculations run from unit start to unit start with the system state being updated prior to each segment.

Example - Segment 7 (16.0h-17.5h)

From Fig. 1.4 it is evident that unit 7 (200 MW) comes on-line at 16h, but since it has a run-up time of 2h it is not able to come on-line until 18h. So, an economic loading calculation is performed from interval 16h to 17.5h with unit 7 unavailable, Fig 1.5. With unit 7 unavailable a less economic commitment of the other units prevails, signified by 60 MW units coming on-line and an increase in the output of larger units.

Run time 0 h 0 min 15 s

Reserve cover = 0.68

Fuel cost (£) = 24685.7

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1050.	0.5	183	0	0	180	74	100	0	169	168	157	0	0	15	0	0	0
1111.	1.0	204	0	0	180	98	100	0	169	168	157	0	0	15	0	0	15
1184.	1.5	225	0	0	180	102	100	0	169	168	157	0	0	15	0	44	19
1224.	2.0	224	0	0	180	102	100	0	169	168	157	0	0	30	0	44	44

Figure 1.5 Segment 7

d. Overall Evaluation

Incorporation of the lead times in this way for the 24-h loading gave the following results:

Segment	Interval	Cost
1	0.5h-1.0h	£9,199.60
2	1.5h-3.0h	£17,711.60
3	3.5h-5.0h	£18,176.60
4	5.5h-8.0h	£22,947.00
5	8.5h-10.0h	£22,533.00
6	10.5h-15.5h	£59,049.10
7	16.0h-17.5h	£24,685.70
8	18.0h-22.0h	£48,874.50
9	22.5h-23.0h	£10,237.20
10	23.5h-24.0h	£9,358.10
TOTAL		**£242,772.40**

Forecasting Credit = £242,772 - £235,279

= £7,493

This is a 3.2% increase in fuel costs from perfect prediction. Similar analysis was carried out for a summer day (Thursday 22nd June 1989), Table 1.3, Fig 1.2 and a 2.0% increase in fuel costs from ideal prediction was observed.

Review of Economic Justification

It has been found that, as expected, fuel costs are reduced significantly by ideal prediction, the main benefit being that thermal unit commitment lead times are correctly incorporated in the scheduling process.

A major advantage of the proposed approach is that various forecasting methods can be evaluated in terms of the proportion of the potential saving, identified previously, which can be achieved.

Having highlighted the important role short-term load forecasting plays in the economic optimisation of power system operation, the next step is to review the general methodology of forecasting.

1.4 Forecasting Methods

A wide variety of forecasting procedures are available, and it is important to realise that no single method is universally applicable - the forecaster must choose the procedure which is most appropriate for a given set of conditions. Forecasting methods can be divided into two basic types - qualitative methods and quantitative methods.

Qualitative methods employ forecasts made on a subjective basis using intuition, commercial knowledge and any other relevant information. The objective is to bring together in a logical, unbiased, and systematic way all the information and judgements which relate to the factors being estimated. Three main qualitative forecasting techniques are frequently used - subjective curve fitting, the Delphi Method, and time independent technological comparisons. Subjective curve fitting is self explanatory and simply involves the use of expert opinion to subjectively construct a curve that reflects the life cycle of the variable of interest. The Delphi Method, on the other hand, uses a physically separate panel of experts to produce predictions concerning a specific problem. The use of time independent technological comparisons is often used in predicting technological change. The method involves predicting changes in one area by monitoring changes that take place in an other area.

Unlike qualitative methods, all quantitative forecasting techniques follow a basic strategy. Past data are analysed to identify a pattern that can be used to describe them. Then this pattern is extrapolated into the future to make forecasts. This strategy relies on the assumption that the pattern has been identified and will continue into the future. Obviously, this assumption is more likely to be valid in the short-term. Quantitative forecasting methods can be grouped into two kinds : time series (univariate) and causal.

A time series is simply a time ordered sequence of observations of a variable. The special feature of time series analysis is the fact that successive observations are usually not independent and that the analysis must take into account the order of the observations. When successive observations are dependent, future values may be predicted from past observations. If a time series can be predicted exactly it is said to be deterministic. However, most time series are stochastic in that the future is only partly determined by past values, so that exact predictions are impossible and must be replaced by the idea that future values have a probability distribution which is conditioned by a knowledge of past values.

Causal models involve the identification of other variables related to the variable to be predicted. Following this, a model is developed that describes the relationship between these exogenous variables and the variable to be predicted (multivariate). A causal model is the most sophisticated form of forecasting tool. It expresses mathematically the relevant causal relationships, and may also directly incorporate the results of a time series analysis.

Various univariate and multivariate models are outlined in the following sections.

1.4.1 Univariate

Simple Exponential Smoothing

Given a non-seasonal time series with no systematic trend, x_1, x_2, ..., x_N, it is natural to take as an estimate of x_{N+1} a weighted sum of the past observations,

$$\hat{x}(N,1) = c_0 x_N + c_1 x_{N-1} + c_2 x_{N-2} + ...$$

where c_i are the weights. Obviously more weight is given to recent observations and less weight to observations further in the past.

This method has the advantages of being quick to prepare and having minimal data requirements. However it has low accuracy and is unable to cope with trend and seasonal patterns in the data.

Holt Winters Forecasting Procedure

This is simply the exponential smoothing procedure generalised to deal with time series containing trend and seasonal variation.

Consider daily observations, allowing L_t, T_t, and I_t to denote the local level, trend and seasonal index respectively at time t. Also let α, γ, δ denote the three smoothing parameters for updating the level, trend and seasonal index respectively. Then when a new observation x_t becomes available, the values of L_t, T_t, and I_t are all updated. If the seasonal variation is multiplicative then the updating equations are:

$$L_t = \alpha(x_t / I_{t-24}) + (1-\alpha)(L_{t-1} + T_{t-1})$$
$$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1}$$
$$I_t = \delta(x_t / L_t) + (1-\delta)I_{t-24}$$

and the forecasts from time t are then

$$\hat{x}(t,k)=(L_t+kT_t)I_{t-24+k}$$

for k=1, 2, ..., 24. Analogous formulae for the additive seasonal case exist.

This method has all the advantages of exponential smoothing but its accuracy is much greater.

Decomposition

If a time series exhibits trend effects and seasonal effects, it can be useful to 'decompose' it to isolate these effects. The multiplicative decomposition model achieves this:

$$y_t=TR_t\times SN_t\times CL_t\times IR_t$$

where

y_t = the observed value of the time series in time period t

TR_t = the trend component (or factor) in time period t

SN_t = the seasonal component (or factor) in time period t

CL_t = the cyclical component (or factor) in time period t

IR_t = the irregular (residual) component (or factor) in time period t

Again there is an analogous formula for the additive case. Although decomposition is easy to understand, and to use, and ideal for dealing with cyclical components, it has no theoretical basis - it is strictly an intuitive approach.

Box-Jenkins Modelling

Univariate Box-Jenkins is a time series modelling process which describes a single series as a function of past values of the series (autoregressive) and/or a combination of previously fitted errors (moving average). The general model is known as a seasonal autoregressive integrated moving average model (SARIMA), expressed by the following equation:

$$\phi(B)y(t) = \theta(B)e(t)$$

where

$$\phi(B) = (1-\phi_1 B - \ldots - \phi_p B^p)(1-\phi_{s1}B^s - \ldots - \phi_{sp}B^{sp})(1-B)^d(1-B^s)^D$$

and

$$\theta(B) = (1-\theta_1 B - \ldots - \theta_q B^q)(1-\theta_{s1}B^s - \ldots - \theta_{sq}B^{sq})$$

where

y(t) = discrete time series

e(t) = white noise sample input

t = sample interval equal to one hour

ϕ_i = autogressive component

θ_i = moving average component

ϕ_{si} = seasonal autoregressive component

θ_{si} = seasonal moving average component

B = backward shift operator such that By(t)=y(t-1)

$(1-B)^d$ and $(1-B^s)^D$ in the $\phi(B)$ polynomial indicate non-seasonal and seasonal differencing with the superscripts d and D indicating the degrees of differencing.

In spite of its complexity the Box-Jenkins method provides the most accurate forecasts in immediate and short-term forecasting. It allows for a wide range of possible models for the data and provides a strategy for selecting a model from that class which best represents the data. In view of the importance of this method in the present work, a detailed explanation will be given in Chapter Two.

Bayesian Forecasting

The Bayesian forecasting procedure uses the Holt-Winters species of model and has only recently been developed. Its accuracy appears to be comparable to that of the Box-Jenkins method but it is computationally complex and difficult to understand.

1.4.2 Causal

Multiple Regression

In the multiple linear regression approach the variable of interest (dependent variable) is linearly related to one or other independent variables called explanatory variables. The model takes the following form:

$$y(t) = a_0 + a_1 x_1(t) + \ldots + a_n x_n(t) + a(t)$$

where

y(t) = dependent variable

a(t) = a random variable with zero mean and constant variance

$x_1(t)$, ..., $x_n(t)$ are explanatory variables correlated with y(t)

a_0, a_1, ..., a_n are the regression coefficients

Explanatory variables of this model are identified on the basis of correlation analysis and the regression coefficients are estimated using the least squares estimation technique.

Although having the inherent advantages of a causal model this approach requires long off-line analysis and the accuracy of the results depend heavily on the model assumed at the beginning.

Multivariate Box-Jenkins

A problem with mixed ARIMA models is that related or explanatory variables are not included explicitly. This can be achieved by using a transfer function model developed by

Box-Jenkins. Since multivariate (transfer function) models combine some of the characteristics of the univariate ARIMA models and some of the characteristics of multiple regression analysis the model blends the time series approach with the causal approach.

The multivariate Box-Jenkins method should produce more accurate forecasts than the univariate Box-Jenkins method, since it uses information about other variables related to the variable being forecasted. However it is more difficult to understand than the univariate method and, being relatively new and innovative, it is not so well proven.

Econometric Models

Econometric models assume that an economic system can be described by a set of simultaneous equations. Economists distinguish between exogenous variables, which affect the system but are not themselves affected, and endogenous variables which interact with each other. The simultaneous equation system involving k independent (endogenous) variables, $\{Y_i\}$, and g predetermined (exogenous) variables $\{X_i\}$, may be written :

$$Y_i=f_i(Y_1,...,Y_{i-1},Y_{i+1},...,Y_k,X_1,...,X_g)+e_i$$

where

$i=1, 2, ..., k$

e_i is the error term

Some of the exogenous variables may be lagged values of Y_i. These equations can be solved to give what is known as the reduced form of the system, namely

$$Y_i=F_i(X_1,...,X_g)+e_i$$

Econometric models are more difficult to understand and more costly than time series models. Also for immediate and short-term forecasts, the Box-Jenkins method is often more accurate than complex econometric methods.

In choosing a forecasting method for use in a given situation many factors, most of which arise from the nature of the associated problem, need to be considered:

Lead-time

Lead-time is simply the length of time into the future for which forecasts are required. Simple exponential smoothing, Holt-Winters, decomposition, and Box-Jenkins modelling are used for the very short-term.

Time to prepare forecasts

This is the total time needed to collect the necessary data, analyse them and prepare the forecast.

Pattern of data

Four basic subpatterns exist, some combination of which usually exists in any data series - trend, horizontal, seasonal and cyclical. A knowledge of the types of subpattern included in the data is important because different types of methods vary in their ability to cope with various patterns.

Data requirements

Various forecasting methods require differing amounts of historical data, so the quantity of data available is vital in selecting the most appropriate method. The accuracy and timeliness of the data that are available is also relevant.

Ease of understanding

The ease with which a forecasting method is operated and understood in relation to its human interface has important implications in the practical application of the method.

Cost

The element of cost is one of the criteria that is traded off against such things as accuracy, ease of application, and the pattern. The costs of forecasting depend very much on the method itself and its inherent complexity as well as on its data requirements and the number of items to be forecast.

Accuracy

Accuracy of a method is the extent to which forecast values approximate to the actual values that emerge in time. The level of accuracy required is determined by the importance of the decision being made and the influence of the forecast on that decision.

There is no 'best' forecasting procedure, the choice of method depends on the application. As this research is essentially interested in the prediction of total system electricity consumption, it is necessary to identify the pattern existing within system load data. The next section examines system load variation to discover if there is an underlying pattern that can be exploited in the preparation of a forecast.

1.5 System Load Variation

The a posteriori analysis and forecast of goods sold by a company constitute a preliminary activity for any technical decision taken, be it of a strategic nature or a tactical one. Within this general context the specificity of an electricity producing company lies in the well known feature of its product: its very low storage nature. It is therefore necessary to analyse very thoroughly the demand, both from the viewpoint of time as well as that of space, in order to design and operate a generation and transmission system able at any moment in time to handle the demand as it occurs, at a cost price as low as possible while ensuring the safety suitable for operation.

System load is the sum of all the individual demands at all the nodes of the power system. The demand or usage pattern of an individual load (device) or customer is quite random and highly unpredictable. This is exacerbated by the very broad diversity of individual usage patterns in a typical utility. Fortunately, however, the totality of individual loads results in a distinct consumption pattern which can be statistically predicted. According to Galiana (3) system load behaviour is influenced by a number of factors, classified into four main categories.

1.5.1 Economic Factors

The economic environment in which the utility operates has a clear effect on the demand consumption pattern. Factors such as the service area demographics, levels of industrial activity, changes in the farming sector, the nature and level of the penetration\saturation of the appliance population, developments in the regulatory climate and more generally, economic trends all have significant impacts on the system load growth trend. These economic factors are not, however, explicitly represented in short-term load forecasting models because of the longer time scales associated with them.

1.5.2 Time Factors

Three principal time factors - seasonal effects, weekly-daily cycle, and legal and religious holidays, play an important role in influencing load patterns. The seasonal changes basically determine whether a utility is summer or winter peaking.

The weekly-daily peaking periodicity of the load is a consequence of the work-rest pattern of the service area population. Fig 1.6 provides examples of typical weekly summer and winter load patterns for the NIE system.

Also the existence of statutory and religious holidays has the general effect of significantly lowering the load values to well below 'normal' - 12th July in Northern Ireland for example.

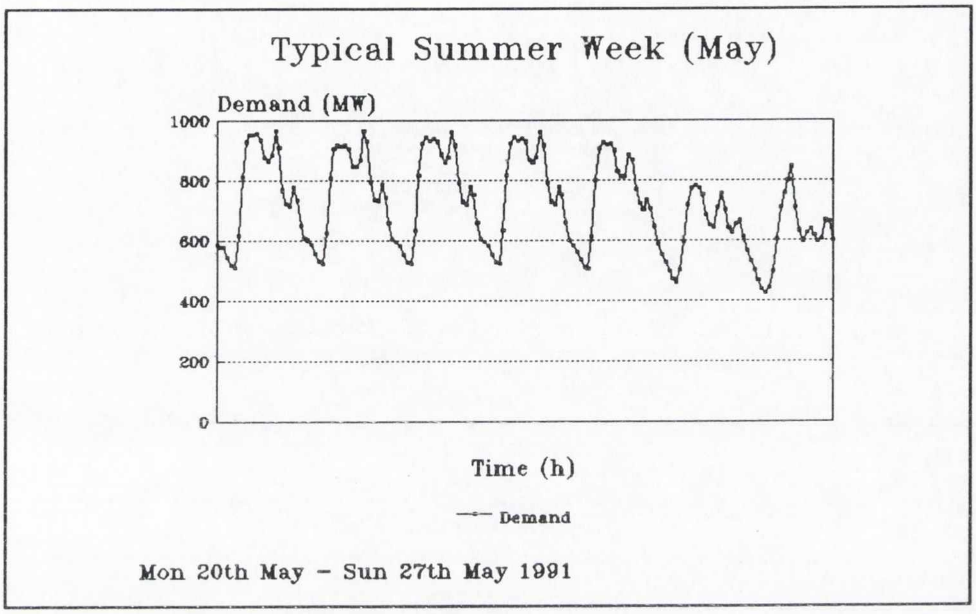

Figure 1.6a Typical Summer Week

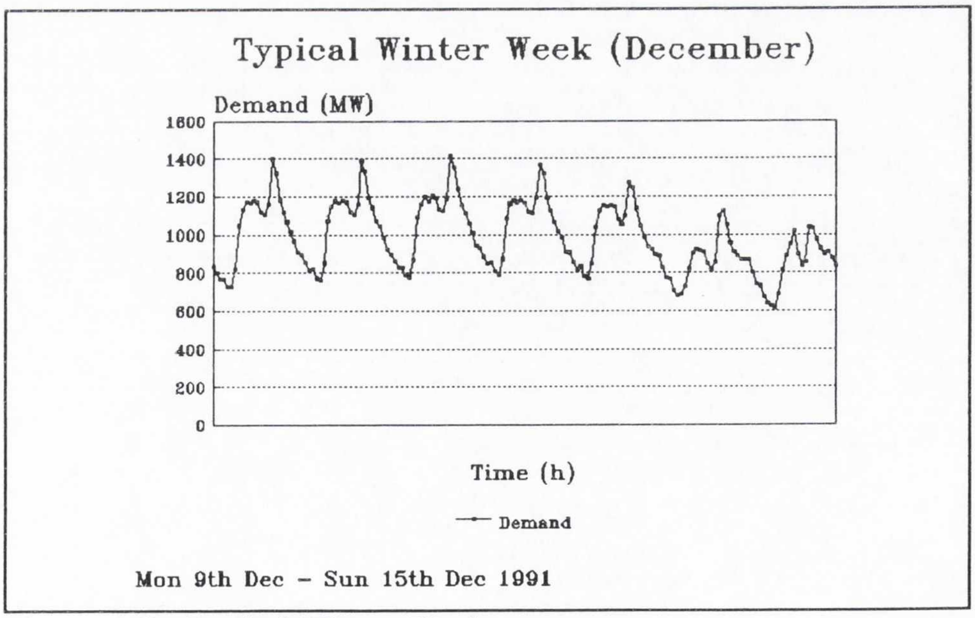

Figure 1.6b Typical Winter Week

1.5.3 Weather Factors

Meteorological conditions are responsible for significant variations in the load pattern. This is true because most utilities have large components of weather sensitive load, such as those due to space heating, air conditioning and agricultural irrigation.

In many systems, temperature is the most important weather variable in terms of its effect on the load. For any given day, the deviation of the temperature variable from a normal value may cause such significant load changes as to require major modifications in the unit commitment pattern. Other factors that impact on load behaviour are wind speed, precipitation, and cloud cover/light intensity.

Also Meslier (4) indicates that weather variations influence the load curve of a given day in two different forms. As mentioned previously, there is an effect on the average level of the load curve i.e. the daily energy consumed. This is most important effect and demands most attention, Fig 1.7. However, weather fluctuations also influence the shape of the load curve.

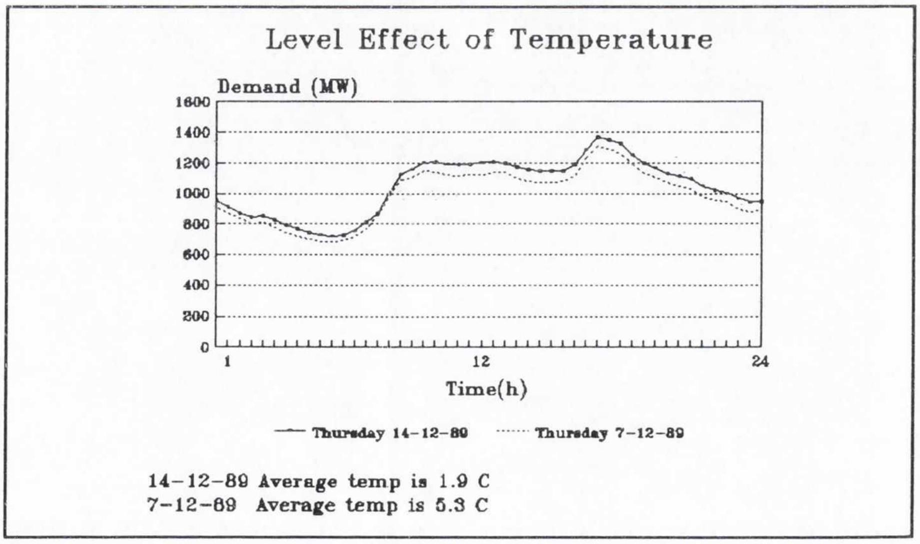

Figure 1.7 Temperature Effect

This effect is less pronounced and mainly due to the development of night space heating. Initial steps in the development of this problem are considered later.

1.5.4 Random Disturbances

A power system is continuously subject to random disturbances reflecting the fact that the system load is a composite of a large number of diverse individual demands. There are also certain events such as widespread strikes, shutdown of industrial facilities, and special television programs whose occurrence is known a priori, but whose effect on the load is uncertain.

So, univariate forecasting, which utilises only past values of load to forecast future values, obviously has problems when there is a sudden change in the weather. The dependent variable of load, which is to be forecasted, is influenced by these independent weather variables. Consequently a multivariate method which includes this causal effect may be required to include weather variables when forecasting the load.

Therefore, neglecting holidays and special effects, forecasting techniques can readily model demand variation, characterised by the seasonal trend, regular rhythmic variation between different days of the week, and the continuous variation of the level of demand with time of day.

Having outlined the pattern existing within system load, the following sections review methods applied to the problem of forecasting in general and short-term load forecasting by other authors.

1.6 Comparative Review of Forecasting Procedures

1.6.1 Review of Short-term Forecasting

The problem of short-term load forecasting has received extensive attention in the past 15 years (5-8). A variety of techniques have been proposed that usually mirror developments in statistical forecasts.

These procedures typically make use of two basic models: peak load models and load shape models. In peak load models the daily or weekly peak load consists of two parts: a weather independent base load and a weather dependent component added to the base load (9-11). Despite their structural simplicity these models are not widely used since they do not provide any information about the shape of the load curve.

Load shape models describe the load as a discrete time series (process) over the forecast interval. They can be categorised into two groups: time-of-day or static models and dynamic models. In static models the load is considered to be a linear combination of explicit time functions, usually sinusoids, exponentials or polynomials. The coefficients of these functions (model parameters) are estimated through linear regression or exponential smoothing analysis applied to a set of past load observations (12-14). A special case of static models is the spectral decomposition model, in which time functions represent the eigenfunctions corresponding to the autocorrelation function of the load time series (15). Even though such time series models are structurally simple they are of limited use since they do not accurately represent weather effects and the stochastically correlated nature of the system load process.

In a dynamic load model the load behaviour is modelled as a dynamic process where the value of the load at any one time depends not only on the time of day, but on the past behaviour of the load, weather, and a random process. They are of two basic types : Box-

Jenkins classification of models and state space models.

The Box-Jenkins classification includes ARIMA models and transfer function models and, according to Moghram and Rahman (7), 'this method appears to be the most popular approach that has been applied and is still being applied to short-term load forecasting in the electric power industry'. They are well suited to the load forecasting problem because of their ability to model multiperiodic and non-stationary processes and because they can allow for explicit temperature inputs (transfer function model). Hagan and Behr (16) and Vemuri (17) have applied the Box-Jenkins methodology and shown that the transfer function model provides greater accuracy than the univariate model (7,18-20).

Several authors prefer to use state equations to model the load demand. The main reason is that they can then use the powerful Kalman filtering theory to obtain the optimum forecasts. However, state-space models require more explanatory variables and parameters than Box-Jenkins modelling.

Almost all of these techniques fall into the realms of statistical techniques. The exception to this is a recent approach by Rahman and Bahmgar (21) which is based on applying a knowledge based algorithm to the load forecasting problem. Some authors have also applied the expert system approach (22,23). Related work on pattern recognition and cluster analysis also exists (24-27). More recently, neural network analysis has been employed to solve the problem of short-term load forecasting (28-31).

1.6.2 Comparative Review

Frequent mention has been made of the comparative merits of various types of forecasting techniques. Makridakis (32,33) compared a number of different univariate extrapolative forecasting techniques and found that when there was considerable randomness in the data, statistically sophisicated methods did not do any better than the simple methods. He also found that seasonal patterns could be predicted just as well by the simple as by the sophisicated techniques. Results produced by Makridakis indicated that the mean absolute percentage error for the Box-Jenkins model was much lower than for any other technique which he studied. Price (34) also compared a number of forecasting methods applied to electricity demand and found that the error for the Box-Jenkins model was much lower than for any other technique which he investigated. Chatfield (35) considered many different types of modelling and forecasting techniques and concluded that there is no one method which is the 'best' in every situation, but rather the best choice depends on the forecasting horizon, the type of data and the manner in which accuracy is measured. Holt-Winters and Box-Jenkins methods were favoured by Chatfield, although he expressed a personal preference for Holt-Winters.

Abu-El-Magd (5) reviewed a number of on- and off-line methods for modelling and forecasting the demand for electrical energy. From Abu-El-Magd's paper the Box-Jenkins approach was found to be the most favourable technique for the required application - the forecasting of electricity demand. Pepper's (36) decision to employ Box-Jenkins techniques rested 'mainly on this method's attempt to take account of the serial and cross-correlation in the identification and estimation of models'. This point is emphasised by Hagan (16). Many other authors have discussed and successfully used the Box-Jenkins approach - McCafferty (37), Dodds (38), Laing (39), Farmer (40), Shuaib (41), Bolzern (42), Barakat (43), Piggot (44), Demirovic (45), McDonald (46), Poysti (47), Karanta (48).

Adiata et al. (49) provided a comparison of practices in regard to demand prediction in England and Wales, France and Italy, concluding that in the short-term prediction methods are converging. All three utilities employ Box-Jenkins based methods.

The univariate Box-Jenkins method which only considers past values of the series being modelled, together with previous errors, is limited in that it does not include external variables in the models. The effect of external variables not being modelled will limit the dynamics of the model and it will only respond to changes in relevant variables when these changes have been reflected in the past values of the series. The Box-Jenkins procedure can however be extended to include external variables, linking the input with the output variables by means of a transfer function, also known as multivariate analysis. Similar routines are used in multivariate modelling as are used in the univariate case and this causal/time series method should therefore be the optimum method to employ.

1.7 Objectives of the Research

Since NIE operates an isolated system, generated electricity must be dispatched to cover total consumer demand, system losses and all operating requirements of the system. At present, system demand is forecasted manually by dispatching engineers by searching for a similar day and by making intuitive adjustments to that day's load to account for differences in weather conditions and other factors that may affect demand. Consequently, although an experienced operator can produce accurate forecasts, a formal procedure is required to produce consistency in results. This research proposes the incorporation of the system identification techniques of Box-Jenkins time series modelling to develop load forecasting models that utilise historical load patterns and the underlying relationship between load and weather in their execution.

Indeed, assessment of the effect of weather variables on short-term demand is a major aim of the research programme. This applies to both the effect of weather on the average level of the load curve and the shape of the load curve. Since the latter influence is due mainly to the development of night space heating, this problem has many further implications which are also considered. Therefore emphasis has been placed on the development of a data acquisition system capable of continuous measurement of meteorological variables and its subsequent installation at a suitable location.

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2.1 Introduction

Box and Jenkins (1) introduced a comprehensive approach to deriving, from time series data, models for prediction, simulation and control of stochastic or random processes. The modelling process relies upon a recursive scheme in which sample 'finger-prints' (autocorrelations and partial autocorrelations) are used to identify a plausible equation. Estimation is then used to evaluate the sufficiency and necessity conditions of the candidate models. This process is repeated until the 'signal' and noise have been completely decoupled providing an 'adequate' model. The singular advantage with Box-Jenkins methods is that they allow the data to 'speak for themselves'.

2.2 Univariate Box-Jenkins Modelling

The univariate model has the advantage of utilising only past data of the series to develop models and forecasts of future demand. This minimises the collection of external data which may prove to be of no significance to model enhancement. The Box-Jenkins model is also known as an AutoRegressive Integrated Moving Average (ARIMA) model, which can be used to model stationary or non-stationary series. The fundamental time series models are the autoregressive and moving average models, with the Box-Jenkins model being a combination of the two.

In an autoregressive (AR) model the current value, y(t), can be expressed as a linear aggregation of previous values of the process, y(t-1) .... y(t-p) and a random shock. Let the values of a process, of order p, at equal spaced intervals t, t-1, ..., t-p be y(t), y(t-1), ... y(t-p). The AR model will be:

$$y(t)=\phi_1 y(t-1)+\phi_2 y(t-2)+....+\phi_p y(t-p)+e(t) \qquad (2.2.1)$$

ϕ_i = the coefficients of the parameters

e(t) = the random shock (noise)

The name of this model is derived from the term 'regression' in the model which relates a dependent variable y to a set of independent variables x_1, x_2, ..., x_p, plus an error term e. In equation 2.2.1, y(t) is regressed on past values of itself, hence the term autoregressive.

In order to write equation 2.2.1 in a more convenient form, the following operators are introduced:

$$By(t)=y(t-1) \qquad B^m y(t)=y(t-m)$$

where

B = the backward shift operator

$$\phi(B)=1-\phi_1 B-\phi_2 B^2-....-\phi_p B^p \tag{2.2.2}$$

Equation 2.2.1 can now be written as:

$$\phi(B)y(t)=e(t) \tag{2.2.3}$$

In a moving average (MA) model the current value of the process, y(t) can be expressed as a linear combination of previous random shocks, e(t) ....e(t-q). If the order of the process is q, then:

$$y(t)=e(t)-\theta_1 e(t-1)-....-\theta_q e(t-q) \tag{2.2.4}$$

where

θ_i = coefficients of the random shock parameters

A similar operator to that used for the AR process equation 2.2.3 can be used to express the coefficients of the MA process, i.e.:

$$\theta(B)=1-\theta_1 B-\theta_2 B^2-....-\theta_q B^q \tag{2.2.5}$$

The moving average model can be more economically written as:

$$y(t)=\theta(B)e(t) \tag{2.2.6}$$

The notation used for identifying an AR model of order p is AR(p) and the notation used for identifying a moving average model of order q is MA(q).

Combining the AR and MA models will give greater flexibility in fitting actual time series. The ARMA model is the most general and theoretically the most complete of the time series models and can be expressed as:

$$y(t)=\phi_1 y(t-1)+....+\phi_p y(t-p)+e(t)-\theta_1 e(t-1)-....-\theta_q e(t-q)$$

or

$$\phi(B)y(t)=\theta(B)e(t) \tag{2.2.7}$$

where the present value, y(t) is an aggregation of past values and past random shocks.

In practice most time series are non-stationary and this variation must be removed in order to fit a stationary model. If a series is non-stationary then differencing can be used to achieve stationarity. This topic is discussed in more detail in Section 2.3.1. If y(t) in equation 2.2.7 is replaced by ∇_o^d y(t), then it is possible to have a model which describes certain types of non-stationary series. This type of model is called an **integrated** model because the stationary model which has been fitted to the differenced data has to be summed or **integrated** to provide a model for the non-stationary data. If

$$w(t)=\nabla_o^d\ y(t)=(1-B^o)^d\ y(t) \tag{2.2.8}$$

where

o = the order of the differencing factor

d = the degree of the differencing factor

then the general autogressive integrated moving average (ARIMA) process is of the form:

$$w(t)=\phi_1 w(t-1)+....+\phi_p w(t-p)+e(t)-\theta_1 e(t-1)-....-\theta_q e(t-q) \tag{2.2.9}$$

By analogy with 2.2.7 it is possible to write 2.2.9 as:

$$\phi(B)w(t)=\theta(B)e(t) \tag{2.2.10}$$

The above process is known as an ARIMA process of order (p,d,q) where p and q are the orders of the autogressive and moving average process and d is the degree of differencing applied to y(t), which in practice is often taken to be 1.

One final complexity to add to ARIMA models is seasonality. In exactly the same manner that consecutive data points might exhibit AR, MA, mixed ARMA, or mixed ARIMA properties, so data separated by a whole season (period) may exhibit the same properties. Indeed, seasonal as well as regular differences may also be required and seasonal autoregressive and seasonal moving average processes may also exist. The ARIMA notation can readily be extended to handle seasonal aspects. If

$$w(t)=\nabla_o^d\ \nabla_s^D\ y(t)=(1-B^o)^d\ (1-B^s)^D\ y(t)$$

where

s = the order of the seasonal differencing factor

D = the degree of the seasonal differencing factor

the resulting model is given by:

$$\begin{aligned}&(1-\phi_1\ B-....-\phi_p\ B^p)(1-\phi_s\ B^s-....-\phi_{sP}\ B^{sP})w(t)\\&=(1-\theta_1\ B-....-\theta_q\ B^q)(1-\theta_s\ B^s-....\theta_{sQ}\ B^{sQ})e(t)\end{aligned} \tag{2.2.11}$$

This model is known as a multiplicative seasonal model of order $(p,d,q)\times(P,D,Q)_s$.

2.3 Basic Concepts in Univariate Model Building

A summary of the approach to model building, which is an iterative procedure, is shown in Figure 2.3.1. The first stage in the approach is to select a suitable class of models from which to choose an appropriate model for the observed time series. This will come from initial analysis and conditioning of the respective time series to be modelled. The next stage is to select an appropriate model from the chosen class of models to fit the time series. This stage may be further subdivided into an iterative procedure of three stages, namely identification, estimation and diagnostic checking. At the identification stage, a tentative model is selected such that the characteristics of a realisation of the tentative model would resemble those of the the observed time series. This is probably the most difficult stage in the model identification because there is no general approach to determine which model to fit the data. If an incorrect model is chosen at this stage it will be identified and discarded at a later stage in the analysis. At the estimation stage, the tentative model is fitted to the observed time series and its parameters are estimated. The initial estimates obtained during the identification stage can now be used as starting values for the parameter estimation algorithms which utilise an iterative procedure to refine the parameters. Finally, diagnostic checks are applied to the fitted model to observe if it adequately represents the observed time series. If no lack of fit is indicated then the model can be used to forecast future values of the observed time series. However if an inadequacy is found in the model, the iterative cycle of identification, estimation and diagnostic checking is repeated until a suitable model is found. Numerous texts and publications (1-6) have described in detail the procedures used for system identification and model building.

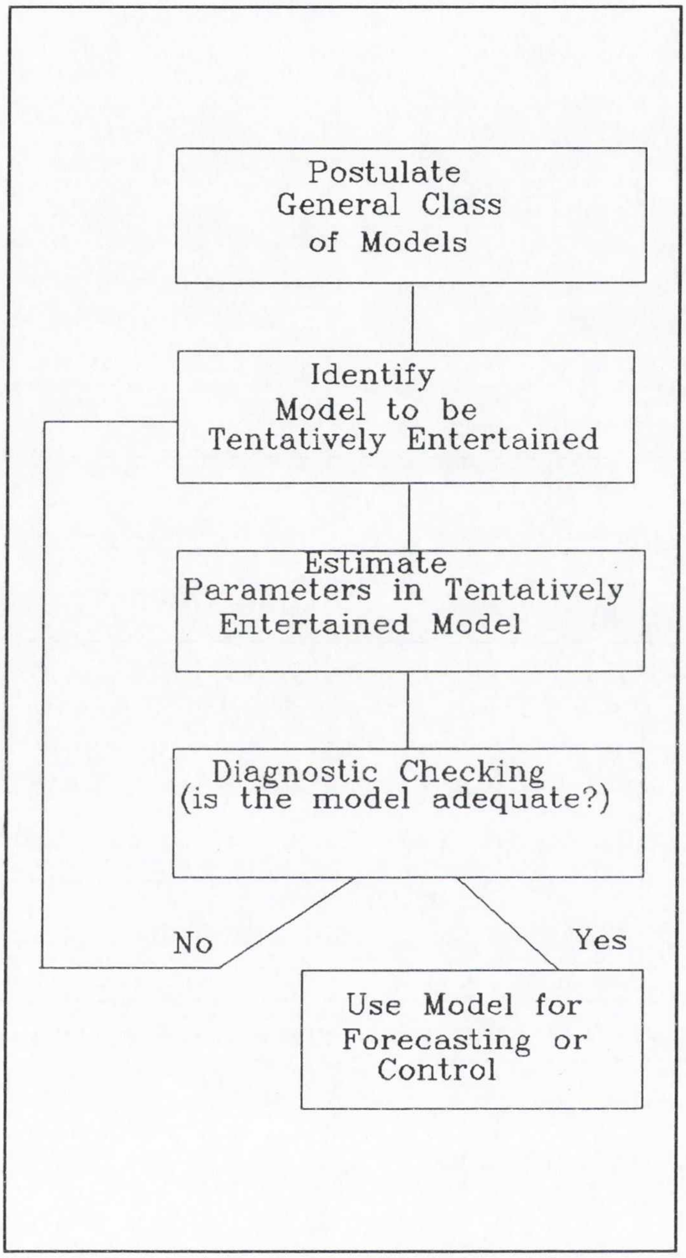

Figure 2.3.1 Approach to Model Building

2.3.1 Identification

Having initially chosen the class of models to use, the next stage is to identify a tentative model. A number of mathematical routines are used as tools in the identification of a tentative model. Initial requirements for time series modelling are that the series to be modelled must be stationary and linear.

It is possible for a time series to be non-stationary in both mean and variance, and appropriate transformations are employed to achieve stationarity. Many of the series which are encountered in different environments exhibit non-stationary behaviour. A time series is said to be strictly stationary if the joint distribution of $Y(t_1)$, ..., $Y(t_n)$ is the same as the joint distribution of $Y(t_1+k)$, ..., $Y(t_n+k)$ for all t_1, ...,t_n, k. In other words, shifting the time origin by an amount τ has no effect on the joint distributions and this holds for any value of n. Therefore:

$$\mu(t)=\mu \tag{2.3.1}$$

and

$$\sigma^2(t)=\sigma^2 \tag{2.3.2}$$

where

μ = the mean, and

σ = the standard deviation

and both constants do not depend on the value of t.

In practice it is often necessary to define stationarity in a less restricted manner than that described above. A process is said to be weakly stationary if its mean is constant and its autovariance function depends only on the lag so that:

$$E[Y(t)] = \mu \tag{2.3.3}$$

$$Cov[Y(t), Y(t+k)] = \gamma(k) \tag{2.3.4}$$

where

E[Y(t)] = the expected value of Y(t), and

Cov = the covariance

If k equalled zero then the above assumption implies that the variance, as well as the mean, is constant. It should also be noted that both the mean and the variance are finite. This weaker form of stationarity is the one which is generally used, as many of the properties of stationary processes depend only on the structure of the process as specified by its first and second moments.

If a process is non-stationary in the mean then an upward or downward trend is present in the observed series. A non-stationary series in the mean can be transformed to a stationary series by a device known as differencing which is the subtraction of one value in the observed series from the other. Consider equation 2.2.10:

$$\phi(B)w(t) = \theta(B)e(t) \tag{2.2.10}$$

where

$$w(t) = \nabla_o^d \; y(t) = (1 - B^o)^d \; y(t)$$

A series can be differenced in a number of ways, namely non-seasonal, seasonal or a combination of both. Consider a series with a non-seasonal differencing of degree 1 and order 1 applied:

$$w(t)=\nabla y(t)=(1-B)y(t)=y(t)-y(t-1)$$

or if the order is 2, then

$$w(t)=\nabla_2 y(t)=(1-B^2)y(t)=y(t)-y(t-2)$$

For seasonal differencing of degree D, using half-hourly data, the order will be in multiples of 48 for daily differencing. Therefore

$$w(t)=\nabla_s^D\ y(t)=(1-B^s)^D\ y(t) \tag{2.3.5}$$

where

D = the degree of seasonal differencing

s = the order of the seasonal differencing

Consider a seasonal difference of degree 1 and order 48, then:

$$\nabla_{48}\ y(t)=(1-B^{48})y(t)=y(t)-y(t-48)$$

If both forms of differencing are applied together, then the differenced series will be:

$$\nabla_s^D\ \nabla_o^d\ y(t)=(1-B^s)^D\ (1-B^o)^d\ y(t)$$

If non-seasonal differencing of degree 1 and order 1 and seasonal differencing of degree 1 and order 48 is required to achieve stationarity then:

$$\begin{aligned} w(t) &= \nabla_{48}^1\ \nabla\ y(t) \\ &= (1-B^{48})(1-B)y(t) \\ &= (1-B^{48})(y(t)-y(t-1)) \\ &= y(t)-y(t-1)-y(t-48)+y(t-49) \end{aligned}$$

The updated version of 2.2.10 can now be written as:

$$\phi(B)\nabla_s^D \nabla_o^d y(t)=\theta(B)e(t) \tag{2.3.6}$$

Figure 2.3.2 shows sketches of non-stationary and stationary time series. Time series analysis assumes that all series are stationary, but also assumes that the appropriate model will be linear, so that the observed series can be represented by a linear function of the present and past values of a purely random process. A time series which is non-stationary in the variance can be transformed into a stationary series by a logarithmic or a square root transformation, or some other suitable transformation. Consider the transformed series to be $y^{\lambda}(t)$, then:

$$y^{\lambda}(t)=\log_e y(t)$$
$$y^{\lambda}(t)=y(t)^{0.5}$$

Equation 2.3.6 can then be written as:

$$\phi(B)\nabla_s^D \nabla_o^d y^{\lambda}(t)=\theta(B)e(t) \tag{2.3.6a}$$

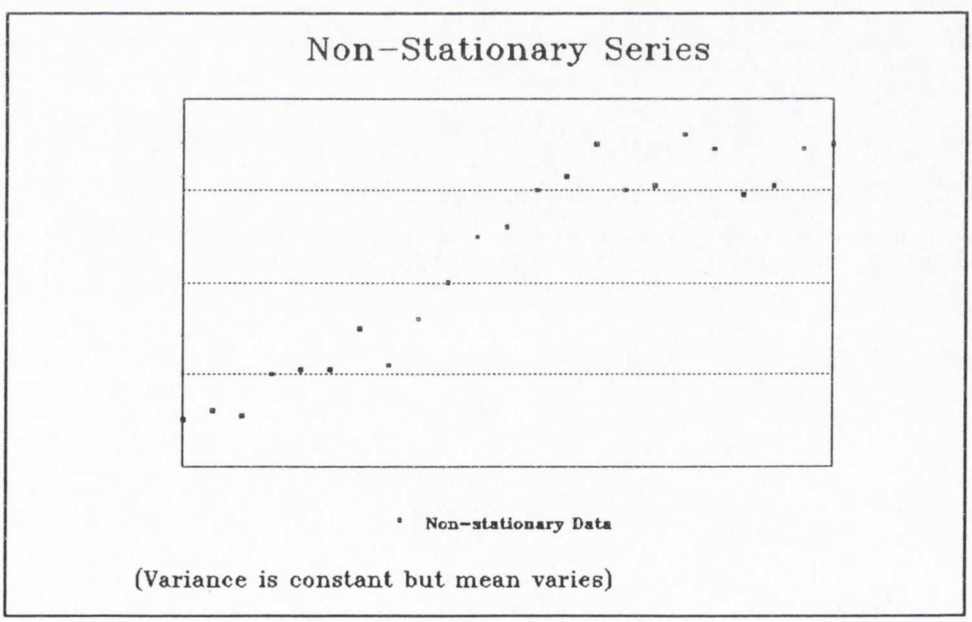

Figure 2.3.2a Non-stationary Series

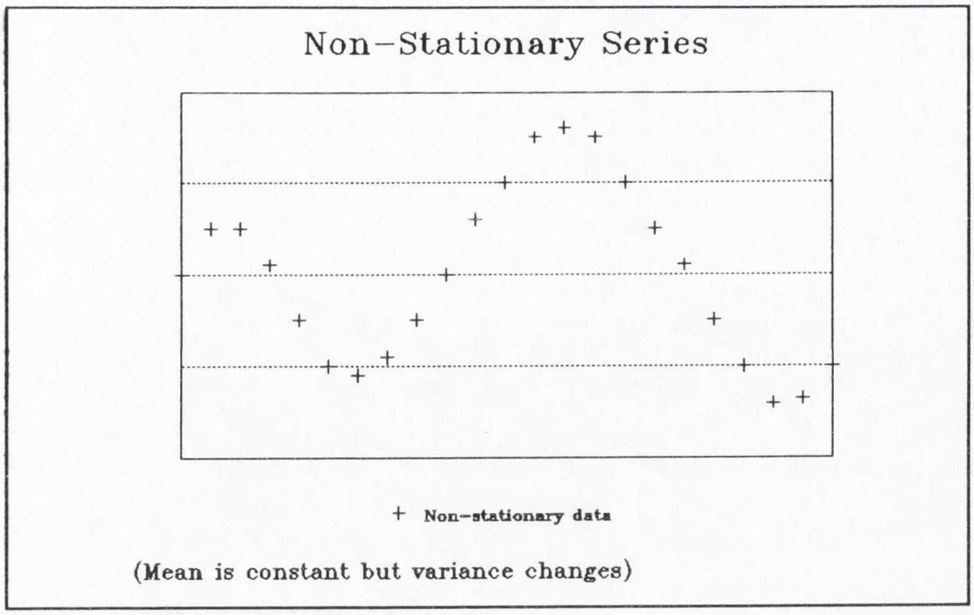

Figure 2.3.2b Non-stationary Series

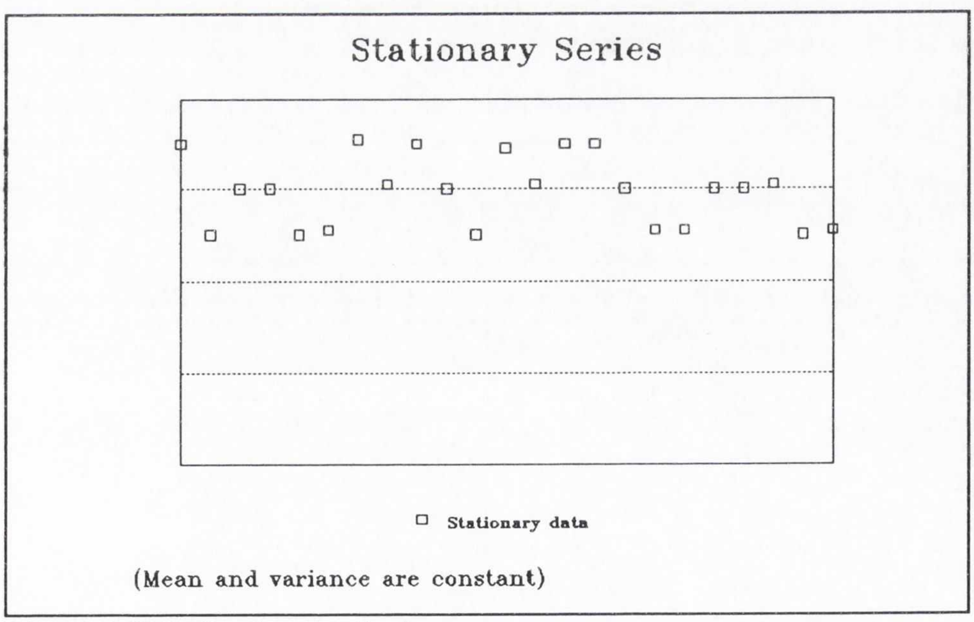

Figure 2.3.2c Stationary Series

The two main statistical tools which are used to identify which type of ARIMA model provides the most adequate representation of the observed time series are the autocorrelation function (ACF) and the partial autocorrelation (PACF). Whilst the ACF is useful in detecting the non-stationarity in the mean, it will not detect non-stationarity due to changes in the variance of the series. This can only be detected by visual examination of a plot of the series, or by calculating the standard deviations of successive subsets of the series.

An initial plot of the raw data would be beneficial since the trends and patterns of the data can be observed, which may be helpful in the identification procedure. In general the theoretical ACF of a moving average model of order q cuts off after a lag of q and the theoretical PACF of an autogressive model of order p cuts off after a lag of p. The theoretical ACF and PACF for an ARMA(p,q) model both tend towards zero.

The visual plot of a time series is often not sufficient to ascertain if the data is stationary or not, but the autocorrelation plot can determine stationarity quite readily. The theoretical autocorrelation plot of a stationary series will fall to zero after the second or third lag, while for a non-stationary series the autocorrelations are significantly different from zero for several time periods. This is because an observation on one side of the mean tends to be followed by a large number of further observations on the same side of the mean because of the trend.

Suppose a stationary stochastic process x(t) has mean μ, variance σ^2, autocovariance function (ACVF) $\gamma(k)$ and ACF $\rho(k)$, then:

$$\rho(k)=\frac{\gamma(k)}{\gamma(0)}=\frac{\gamma(k)}{\sigma^2} \qquad (2.3.7)$$

k = the lag time

(Note that $\rho(0)=1$)

The ACF is an even function in that the correlation between x(t) and x(t+k) is the same as the correlation between x(t) and x(t-k). It gives a measure of the correlation between observations at different lags. At lag k the theoretical ACF is denoted by ρ(k) with r(k) as the estimate, i.e. r(k)=$\hat{\rho}$(k). Note also that:

$$c(k)=\hat{\gamma}(k)$$
$$s^2(k)=\hat{\sigma}^2(k)$$

From equation 2.3.7 the ACF was derived from the ACVF, which is given by:

$$\gamma(k)=\sum_{t=1}^{N-k}\frac{(x_t-\bar{x})(x_{t+k}-\bar{x})}{N} \tag{2.3.8}$$

where γ(k) is the theoretical autocovariance function at lag k, and N is the number of values in the series. Therefore

$$\rho(k)=\frac{\gamma(k)}{\gamma(0)}$$
$$\rho(k)=\frac{\sum_{t=1}^{N-k}(x_t-\bar{x})(x_{t+k}-\bar{x})}{\sum_{t=1}^{N}(x_t-\bar{x})^2} \tag{2.3.9}$$

The sample ACF is of vital importance in this identification procedure, but on its own it does not allow a particular model to be specified. Another important statistical parameter which is used is the sample partial autocorrelation function (PACF). Partial autocorrelations are used to measure the degree of association between x_t and x_{t-k} when the effects of other time lags - 1, 2, 3, ..., up to k-1 are somehow partialled out. This can be demonstrated by considering three observations: x(t), x(t+1), x(t+2). If these observations have a high autocorrelation coefficient at lag 1, then x(t) and x(t+2) will also show a relatively high correlation. Consider ρ(1) as the autocorrelation coefficient at lag 1, then

$$x(t+1)=\rho(1)x(t)$$
$$x(t+2)=\rho(1)x(t+1)$$
$$=\rho^2(1)x(t)$$

If $\rho(1)$ is close to ± 1, then x(t) is highly correlated with x(t+2). Therefore the partial correlation coefficient at lag 2 will reveal whether this correlation between x(t) and x(t+2) is due to the fact that it is correlated or if it is because x(t) and x(t+2) are correlated with x(t+1). The theoretical partial autocorrelation coefficient is denoted by ρ(kk) and the sample partial autocorrelation coefficient by r(kk). The equations for r(kk) are given by:

$$r(11)=r(1) \qquad r(22)=\frac{r(2)-r(2)^2}{1-r(1)}$$

More complicated equations exist for r(33), r(44) etc. - Box and Jenkins (1) detail these equations.

Note that the sample autocorrelation and partial autocorrelation coefficients are only estimates of the theoretical autocorrelation and partial autocorrelation coefficients. The theoretical ACF of a stationary process tends either to die down quickly towards zero with increasing lag k or to cut off after a particular lag k=q. This means that

$$\rho(k)=0, \qquad \text{for } k>q$$

Because the sample ACF r(k) is only an estimate it will probably be small but not zero for k>q. The criterion which is employed to establish whether the sample ACF or the sample PACF are significant at different lags is called the test of significance. This test can determine if the theoretical coefficient is significantly different from zero. For a moving average model MA(q) of order q, the ACF cuts off after lag q, that is ρ(k)=0 for all k>q. Thus for the MA(q) model the sample autocorrelations should all be near zero for k>q. It may be shown, O'Donovan (2), that if the theoretical autocorrelations are zero for k>q, then the sample autocorrelations with k>q are approximately normally distributed with zero mean and a standard deviation that may be estimated by:

$$S=\frac{1}{N^{0.5}}\left[1+2\sum_{i=1}^{q}r^2(i)\right]^{0.5} \tag{2.3.10}$$

If a variable X is normal with mean μ and standard deviation σ, then the probability that a random value of X lies between the limits $\mu \pm (1.96)\sigma$ is 0.95. Applying this to the sample autocorrelation it can be seen that if the autocorrelation r(k) lies outside the limits

$$\pm\frac{1.96}{N^{0.5}}\left[1+2\sum_{i=1}^{q}r^2(i)\right]^{0.5}$$

it is significantly different from zero and it can be concluded that $\rho(k) \neq 0$. Otherwise, r(k) is not significantly different from zero and hence $\rho(k)=0$. An equivalent procedure is to calculate the t-ratio, which is:

$$t_r(k)=\frac{r(k)}{S} \tag{2.3.11}$$

The t-ratio must lie outside the limits ± 1.96 to be significantly different from zero.

A similar test of significance is performed on the sample partial autocorrelations. For an AR model of order p, the PACF cuts off after lag p, i.e. $\rho(kk)=0$ for all $k>p$. If the sample partial autocorrelation coefficients are zero for $k>p$, then the sample partial autocorrelation with $k>p$ is approximately normally distributed with zero mean and standard deviation $1/N^{0.5}$. Hence, if $\rho(kk)=0$ for all $k>p$, the sample partial autocorrelation, r(kk), with $k>p$ should lie between the limits $\pm 1.96/N^{0.5}$ with probability 0.95.

The t-ratio for the partial autocorrelation is identical to that for the autocorrelation, i.e:

$$t_r(kk) = \frac{r(kk)}{s.d} = \frac{r(kk)}{\dfrac{1}{N^{0.5}}} \tag{2.3.11a}$$

So as well as indicating the stationarity of a time series, the autocorrelations and partial autocorrelations of the appropriately differenced series possess patterns which are associated with particular model forms, i.e., by examining the information in the ACF and PACF a first pass conjecture about autoregressive and/or moving average operators to be included can be ascertained. The characteristic properties of tentative ARIMA models are summarised in Table 2.3.1.

Operator	**Theoretical ACF of stationary differenced time series**	**Theoretical PACF of stationary differenced time series**
Regular autoregressive of order 1 $(1-\phi_1 B)$	Tails off towards zero	Cuts off after lag 1
Regular autoregressive of order 2 $(1-\phi_1 B-\phi_2 B^2)$	Tails off towards zero	Cuts off after lag 2
Regular moving average of order 1 $(1-\theta_1 B)$	Cuts off after lag 1	Tails off towards zero
Regular moving average of order 2 $(1-\theta_1 B-\theta_2 B^2)$	Cuts off after lag2	Tails off towards zero
Mixed autoregressive moving average	Irregular pattern at lags 1 to q, then tails off towards zero	Tails off towards zero

Table 2.3.1 Characteristic Properties of Regular Operators

Evidence of a seasonal pattern should also be sought in the sample ACF. A time series is said to have a seasonal pattern if it exhibits a regular pattern which repeats itself after a certain number of basic time intervals. The number of time intervals after which the pattern recurs is known as the **period** of the seasonal pattern and is denoted by **s**. Usually seasonal peaks and troughs can be spotted by visual examination of a plot of the time series but if the time series fluctuates considerably or if trend is present, the seasonal trend may not be visual and may only be detected from the sample ACF.

If there is a seasonal pattern with a period of s time periods, a high positive correlation exists between observations that are s time periods apart and so the sample ACF should exhibit peaks at the seasonal lags s, 2s, 3s etc..

In the case of peak electrical load for the NIE system (week-day only, Sept.-Dec. 1991), Figure 2.3.3, the strong trend in the time series obscures the seasonal pattern in the ACF, Figure 2.3.4. So, because trend is present one regular difference is taken and the seasonal pattern (period 5) is clearly seen , Fig. 2.3.5.

Also if the sample autocorrelations at the seasonal spikes appear to die down very slowly towards zero or are all approximately the same size , then the time series is said to be non-stationary in the seasonal pattern - see Fig.2.3.6, which refers to hourly demand data. This can be removed by taking a seasonal difference of the time series.

Having induced stationarity the properties of regular AR(p) and regular MA(q) models are used to identify the type and order of the regular operator to be included in the tentative model. Then to determine the type and order of the seasonal operator the spikes at the seasonal lags are examined and the characteristic properties of seasonal autoregressive and moving average models, Table 2.3.2, used to identify the type and order of the seasonal operator to be included in the model.

Operator	Theoretical ACF of stationary differenced time series	Theoretical PACF of stationary differenced time series
Seasonal autoregressive of order 1 $(1-\phi_s B^s)$	Autocorrelations at lags s, 2s, 3s tail off towards zero	Partial autocorrelations at lags s, 2s, 3s cut off after lag s
Seasonal autoregressive of order 2 $(1-\phi_s B^s-\phi_{2s} B^{2s})$	Autocorrelations at lags s, 2s, 3s tail off towards zero	Partial autocorrelations at lags s, 2s, 3s cut off after lag 2s
Seasonal moving average of order 1 $(1-\theta_s B^s)$	Autocorrelations at lags s, 2s, 3s, cut off after lag s	Partial autocorrelations at lags s, 2s, 3s tail off towards zero
Seasonal moving average of order 2 $(1-\theta_s B^s-\theta_{2s} B^{2s})$	Autocorrelations at lags s, 2s, 3s, cut off after lag 2s	Partial autocorrelations at lags s, 2s, 3s tail off towards zero

Table 2.3.2 Characteristic Properties of Seasonal Operators

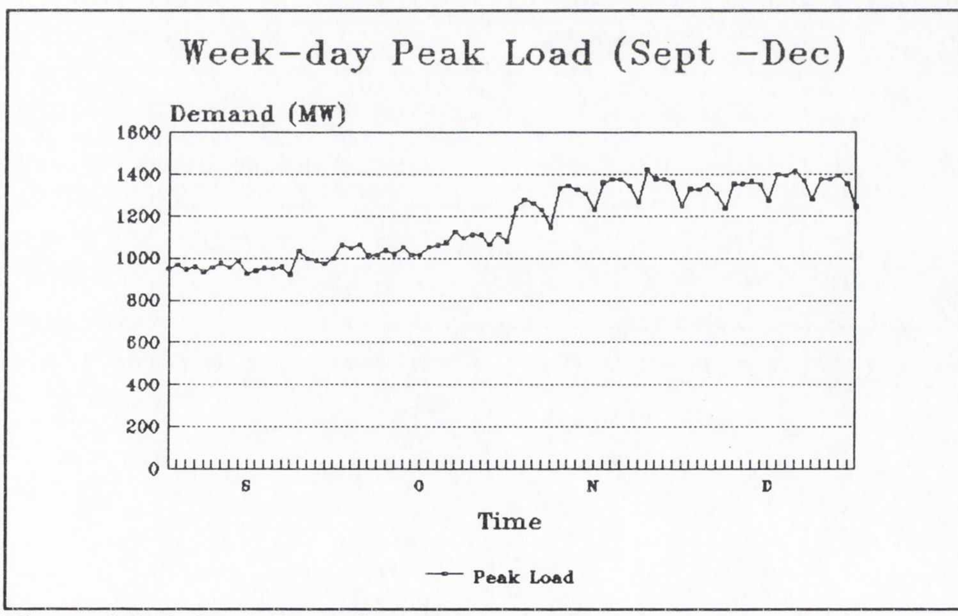

Figure 2.3.3 Peak Load Sept-Dec

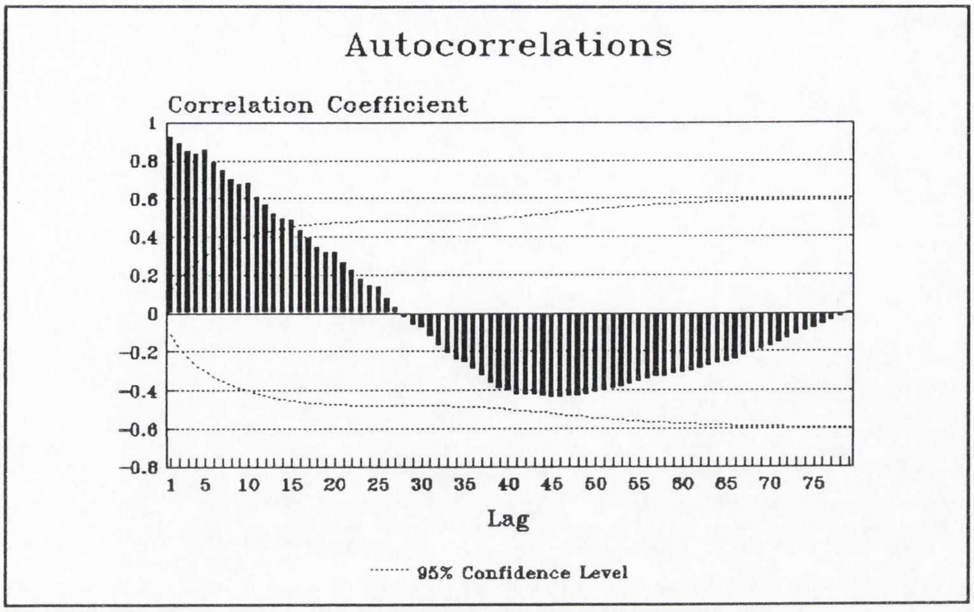

Figure 2.3.4 Autocorrelations (no differencing)

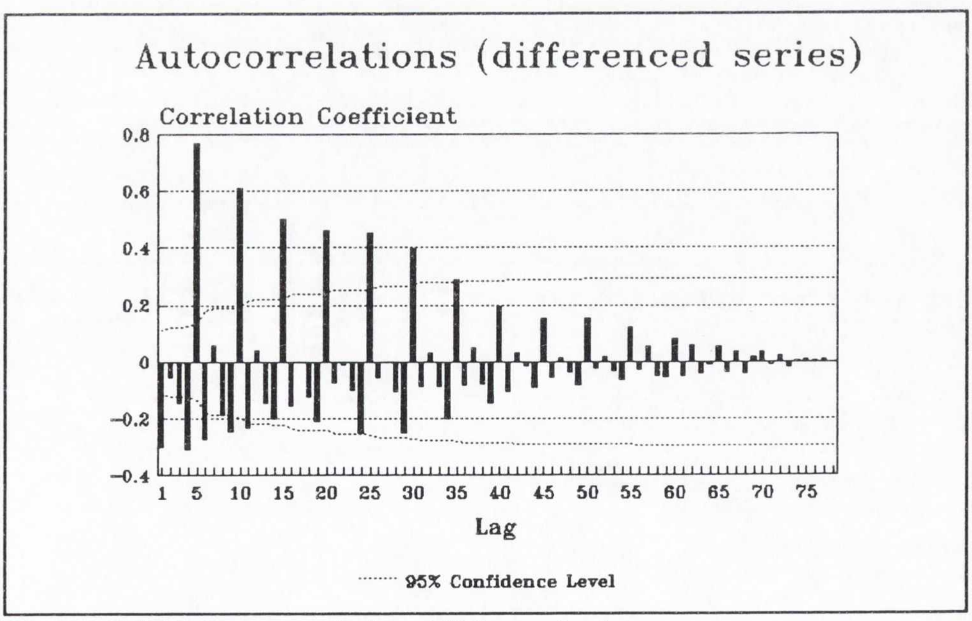

Figure 2.3.5 Autocorrelation (differencing)

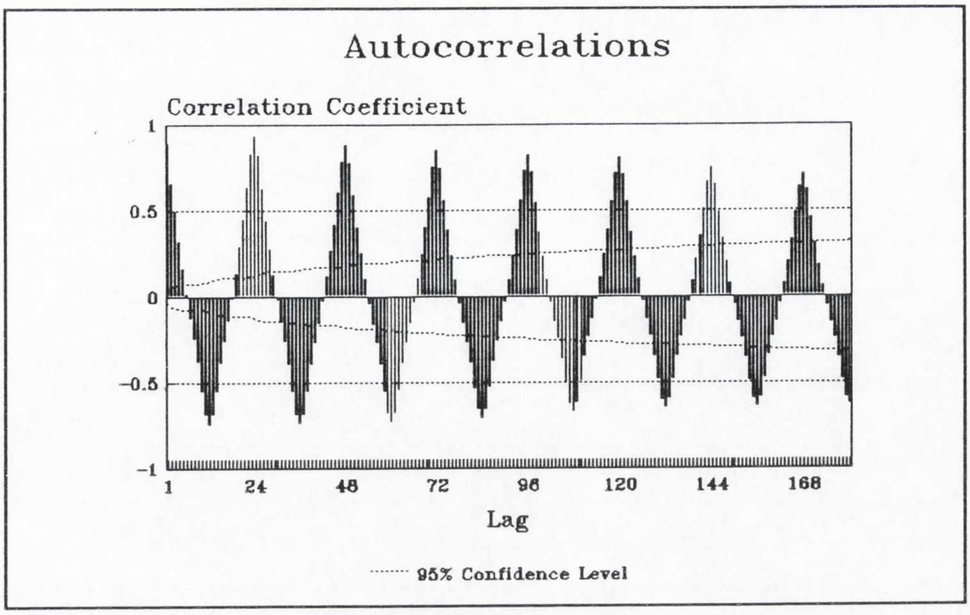

Figure 2.3.6 Seasonal Non-stationarity

2.3.2 Goodness of Fit

The ACF and PACF will provide an approximation of the significant parameters of the model but it is usually difficult to assess the true order of a process from these functions alone. For processes with a number of significant parameters, the ACF may be a mixture of damped exponential or sinusoidal functions and it is difficult to identify the model order. Model order is taken to mean the total number of parameters, or the number of input, output or noise terms as appropriate. An effective and widely used method for order determination is to compare the goodness of fit of the predicted data obtained from the tentative model with the observed data for models with progressively higher orders. The goodness of fit is measured by the sum of the squares of the residuals J, where:

$$J=(y-X\hat{\theta})^T(y-X\hat{\theta}) \tag{2.3.12}$$

where

$\hat{\theta}$ = the least squares parameter estimate for a given model order n y = the dependent variable

X = the combination of past values and errors

In general, J decreases as the model order increases, however the reduction of J ceases to be significant when n becomes greater than the true value of the model n_o. The appropriate model order can be chosen as the one for which J ceases to decrease significantly.

Another approach used for model order determination is based on Akaike's Final Prediction Error (FPE) criterion and is concerned with comparing processes of different orders, where:

$$FPE = \left[\frac{(N+p)}{(N-p)}\right]\sigma_e^2 \tag{2.3.13}$$

where

N = number of observations used for fitting

p = number of parameters in the model

σ_e^2 = variance of the residual errors

The order p is essentially selected so as to get the estimated one-step-ahead predictor with the smallest mean square error. A more general criterion is to minimise the Akaike Information Criterion (AIC), which is given by:

$$AIC=(-2)\log_e(\text{maximised likelihood})+2(\text{no. of independent parameters estimated}) \tag{2.3.14}$$

Typically, the value of AIC decreases quickly as the number of parameters being estimated is increased, and then increases almost linearly when too many redundant parameters are included. The set of parameters which produce the minimum AIC is chosen as the final estimate. The AIC and Akaikes's final prediction error (FPE) are connected by the equation:

$$AIC = N\log_e(FPE) \tag{2.3.15}$$

Akaike has also developed a Bayesian modification of AIC, known as the Bayesian Information Criterion (BIC), which penalises models with large numbers of parameters. If the number of independent parameters is denoted by p, and N denotes the number of observations to which the model is fitted, then:

$$\mathrm{BIC} = (-2)\log_e(\text{maximised likelihood}) + (p + p\log_{10}N) \tag{2.3.16}$$

Although there is no universal panacea for determining the order of a time series model from empirical data, the above procedures undoubtedly aid the identification procedure.

2.3.3 Univariate Model Parameter Estimation

When the order of the tentative model has been established, the next stage is to obtain estimates of the coefficients of the parameters. There are numerous algorithms which have been used with varying degrees of success to provide parameter estimations with the least squares estimation (LSE) and the maximum likelihood estimation (MLE) algorithms being the most widely used. Indeed, the majority of other algorithms are based on variations of the LSE and the MLE techniques. The LSE is the most popular because of its relatively straightforward understanding. The algorithms used in this research employ a non-linear based variation on the LSE technique, see Section 4.2. A description of the initial estimates of the parameters for these algorithms is now described.

The estimation algorithms employ an iterative process which terminates when the estimated parameter values have converged. The speed of convergence will be significantly enhanced by good initial estimates of the parameters. Initial estimations of an AR(k) process are obtained from:

$$\rho(k)=\phi_1\rho(k-1)+\phi_2\rho(k-2)+....+\phi_k \tag{2.13.17}$$

which is derived in Appendix 2. An example of this estimation can be illustrated by considering an AR(2) model.

$$\rho(1)=\phi_1+\phi_2\rho(1)$$
$$\rho(2)=\phi_1\rho(1)+\phi_2$$

Replacing $\rho(1)$ and $\rho(2)$ with r(1) and r(2), and solving for ϕ_1 and ϕ_2 gives preliminary estimates:

$$\hat{\phi}_1=\frac{r(1)[1-r(2)]}{1-r^2(1)} \qquad \hat{\phi}_2=\frac{r(2)-r^2(1)}{1-r^2(1)}$$

For the MA(q) process the theoretical autocorrelations can be expressed in terms of the MA coefficients as follows:

$$\rho(k)=\frac{-\theta_k+\theta_1\theta_{k+1}+....+\theta_{q-k}\theta_q}{1+\theta_1^2+....+\theta_q^2}\ , \quad k=1,2,....q \tag{2.3.18}$$
$$=0\ , \qquad k>q$$

and the derivation is given in Appendix 2. Since the theoretical values ρ(k) are unknown, preliminary estimates of the coefficients θ_1, θ_2, ..., θ_q can be obtained by substituting empirical autocorrelations r(k) into 2.3.18 and solving. Considering a MA(2) process, with q=2, from 2.3.18, the autocorrelations are:

$$\rho(1)=\frac{-\theta_1(1-\theta_2)}{1+\theta_1^{\,2}+\theta_2^{\,2}} \qquad \rho(2)=\frac{-\theta_2}{1+\theta_1^{\,2}+\theta_2^{\,2}} \qquad \rho(k)=0,\ k\geq 3$$

Substituting r(1) and r(2) for $\rho(1)$ and $\rho(2)$ yields two equations and two unknowns, θ_1 and θ_2, which must be solved by an iterative process (1). Initial estimates for a mixed ARMA model may also be obtained.

Once initial estimates have been made, the next step is to obtain the correct coefficients of the parameters. In equation 2.3.19 the objective is to select the coefficients of the parameters of the function f(t), such that e(t) is a zero-mean white noise sequence of minimum possible variance.

$$y(t)=f(t)+e(t) \tag{2.3.19}$$

Backcasting is used to enhance parameter estimations of an adjusted series. The majority of parameter estimation algorithms assume that the predicted values are the actual values (zero) for the lost observations of a model, thus the residuals are zero. For example, the model:

$$(1-\alpha_1 B)y(t)=e(t) \tag{2.3.20}$$

loses one observation and it is assumed that the first fit value is the actual value and that the first residual is zero. So the backcasting method forecasts backwards to obtain better estimates for the lost observations. This procedure has been outlined by Box and Jenkins (1) and O'Donovan (2).

2.3.4 Diagnostic Checking

After having estimated the parameters of a tentatively identified model, it is necessary to perform diagnostic checking to verify that the model is adequate. There are three basic diagnostic checks that must be performed on the estimated model. These tests are for necessity, invertibilty and sufficiency. Each parameter included in the model should be statistically significant (necessary) and each factor must be invertible. In addition, the residuals from the estimated model should be white noise (model sufficiency).

Invertibilty: is determined by extracting the roots from each factor in the model and this condition is independent of the stationary condition. To illustrate the idea of invertibility, consider the model,

$$y(t)=(1-\theta B)e(t) \tag{2.3.21}$$

where the observed y(t) is expressed as a linear combination of random shocks, or previous errors, e(t).

$$e(t)=(1-\theta B)^{-1}y(t)$$

$$e(t)=y(t)+\theta y(t-1)+\theta^2 y(t-2)+.... \tag{2.3.22}$$

$$y(t)=-\theta y(t-1)-\theta^2 y(t-2)-....+e(t) \tag{2.3.23}$$

However, if $|\theta| \geq 1$, the weights diverge in the expansion equation 2.3.22. Thus, at time t, the observed value in equation 2.2.23 depends on y(t-1), y(t-2), ... y(t-n), with weights that increase as n increases. This situation is avoided by requiring that $|\theta| < 1$. Invertibilty is satisfied if the series

$$\pi(B)=(1-\theta B)^{-1}=\sum_{n=0}^{\infty}\theta^{i}B^{n} \tag{2.3.24}$$

converges for | B | , that is, on or within the unit circle.

Statistical significance: The t-ratio can be used to establish if the parameter estimates are statistically significant - significantly different from zero.

$$t\text{-}ratio=\frac{parameter\ estimate}{estimated\ standard\ errors} \tag{2.3.25}$$

If the t-ratio lies outside the limits ± 1.96, then the parameter estimate is assumed to be significantly different from zero, otherwise it is assumed to be zero.

After the preliminary checks on the parameters have been carried out, the main tests of model adequacy are based on a study of the residuals of the fitted model. If the correct model has been fitted, then the observed value of the one step ahead forecast error e(t) is:

$$e(t)=y(t)-\hat{y}(t) \tag{2.3.26}$$

e(t) = white noise

y(t) = actual value

$\hat{y}$(t) = predicted value

The observations in a white noise process are independent and normally distributed with the sample mean zero, and the sample variance σ_e^2, sometimes written as $N(0,\sigma_e^2)$. The residuals are then analysed to determine if they possess these properties. Initially the residuals are plotted against time and if the plot indicates that the variability of the residuals increases with time, then the residuals are assumed not to be white noise and the observed time series is not stationary in the variance. The characteristic feature of a white noise process is that all the theoretical autocorrelations, and hence all the theoretical partial autocorrelations, are zero. Thus, for a residual series to resemble white noise, the sample ACF and PACF should be close to zero. The criterion which was used in the identification procedure in Section 2.3.1 to determine if the ACF and PACF were significantly different from zero also applies here.

Even if one or more of the correlation coefficients are significant, it should not immediately be concluded that the model is inadequate. It is advisable to carry out some further tests and analysis on the tentative model before any decisions are made. One such test is a portmanteau lack-of-fit statistic which is used to detect time series model mis-specification and was developed by Box and Pierce. The test can be used to analyse the first k values of the correlogram of a series of N observations and test whether the residuals are small enough for the residuals to ressemble a realisation of a white noise process.

$$Q=N\sum_{l=1}^{k}\left(r_l(\hat{a})\right)^2 \tag{2.3.27}$$

$r_l(\hat{a})$ = *the sample autocorrelation of lag l for the residuals*
Q = *the Box-Pierce χ-square statistic*

The larger the sample autocorrelations are, then the larger Q will be and vice versa, so that if Q is too large, this indicates that the autocorrelations $r_l(a)$ are too large for the residuals to ressemble a white noise process. If the fitted model is adequate, then Q should be approximately distributed as χ^2, a chi-squared distribution with (m-p-q) degrees of freedom, where p and q are the number of AR and MA terms respectively in the model. From tables of the chi-squared distribution it is possible to find a critical value such that the probability that a random value of Q exceeds this critical value is 0.05. If the calculated value of the Q statistic exceeds the critical value determined from the chi-squared tables, then the value of Q is deemed to be too high and the fitted model is assumed to be inadequate - otherwise the model is assumed to be appropriate. A modification to the Box-Pierce statistic is the Ljung-Box statistic:

$$Q = N(N+2)\sum_{l=1}^{k} \frac{(r_l(\hat{a}))^2}{N-l} \tag{2.3.28}$$

This statistic tests the adequacy of the overall model in exactly the same way.

A study of the sample autocorrelations of the residuals may indicate model inadequacies and suggest that a new parameter be included in the model. On other occasions however, even though diagnostic tests have not detected any inadequacies in the fitted model, it is advisable to consider including a further parameter in the model in the hope that the new model is a better fit than the old one. This is known as overfitting. When the new model is fitted, if the parameter estimate of the new parameter is not significantly different from zero, then the term corresponding to the new parameter may be worth including in the new model. In this case, the estimates of the other parameters in the new models will be similar to the corresponding estimates in the old model. To determine if the new tentative model is a better fit than the old tentative model, the residual variance for each model is calculated. If the residual mean is near zero and N is large, the residual variance is approximately equal to the following expression:

$$\frac{1}{N}\sum_{t=1}^{N} (\hat{a}(t))^2 \tag{2.3.29}$$

In comparing two models the one with the smaller residual variance is preferable.

Various combinations of these criteria are used for diagnostic checking and it should be noted that they are only to be used as guides in obtaining the optimum model. It may be possible to identify from the model building procedure two or more models which can adequately represent the system. Then, according to the **principle of parsimonious parameterisation** the model with the fewest parameters is chosen.

With the optimum model now selected it is possible to obtain forecasts for future values of the observed process.

2.3.5 Forecasting

Consider a time series of t observations, denoted by $z_1, z_2, \ldots, z_t$. These are regarded as observed values of the random variable $Z_1, Z_2, \ldots, Z_t$. A forecast of the time series l time periods in the future is denoted by z_{t+l} and is a random value of the variable Z_{t+l}. Of course, z_{t+l} is currently unknown, but a forecast of z_{t+l}, based on the observed values $z_1, z_2, \ldots, z_t$ is denoted by $\hat{z}_t(l)$. This forecast is said to have origin t and lead time l.

It has been shown (1) that the forecast of z_{t+l} which has the the smallest mean square error is the expected value of the variable Z_{t+l}. This expected value is denoted by

$$E_t[Z_{t+l}]$$

Thus $\hat{z}_t(l)=E_t[Z_{t+l}]$ is the **minimum mean square error** forecast of z_{t+l}, conditional on the information at origin t.

A general ARMA model may be defined in the form of a difference equation as follows:

$$Z_t=\phi_1 Z_{t-1}+\ldots+\phi_p Z_{t-p}+A_t-\theta_1 A_{t-1}-\ldots-\theta_q A_{t-q}$$

Forecasting future values of this model involves very simple calculations and the method by which these forecasts are evaluated applies to all model forms. The method for calculating the forecast $\hat{z}_t(l)$ may be summarised as follows:

a. Using the definition of the model under study, express Z_{t+1} in terms of the variables Z_t, Z_{t-1}, ..., Z_1 (corresponding to the observations z_t, z_{t-1}, ...,z_1 that have occurred at time t), the variables Z_{t+1}, Z_{t+2}, ... (corresponding to the observations z_{t+1}, z_{t+2}, ... that have not occurred at time t), the random shocks A_t, A_{t-1}, ... (which have occurred at time t), and the random shocks A_{t+1}, A_{t+2}, ... (which have not occurred at time t).

b. Since

$$\hat{z}_t(l) = E_t[Z_{t+l}]$$

by taking the expected value of each of these components, conditional on the information at origin t, an expression for $\hat{z}_t(l)$ is arrived at by making the following substitutions in the original expression for Z_{t+1}:

1. Replace Z_t, Z_{t-1}, ... by their observed values z_t, z_{t-1}, ...

2. Replace Z_{t+1}, Z_{t+2}, ... by the forecasts $\hat{z}_t(1)$, $\hat{z}_t(2)$, ...

3. Replace A_t, A_{t-1}, ..., A_1 by the observed one step ahead forecast errors,

$$(z_t - \hat{z}_{t-1}(1)),\ (z_{t-1} - \hat{z}_{t-2}(1))\ ,...,\ (z_1 - \hat{z}_0(1))$$

4. Replace A_{t+1}, A_{t+2}, ... by zeros.

5. In starting the forecasting process, it may be necessary to assume that a_0, a_{-1}, ..., the realisations of the random shocks A_0, A_{-1}, ..., are all zero. Then the forecast $\hat{z}_t(l)$ is built up recursively from the forecasts $\hat{z}_t(1)$, $\hat{z}_t(2)$, ..., $\hat{z}_t(l-1)$.

The future values of a time series cannot be forecast with certainty. The uncertainty in a forecast can be described by the probability distribution of the future value Z_{t+l} at a lead time l. This conditional distribution is normal with mean $\mu(l)$ and standard deviation $\sigma(l)$. So, the actual observation, when it comes to hand, will have a probability of approximately α of lying between the probability limits,

$$\mu(l) \pm u_{\alpha} \sigma(l)$$

where the values $\pm u_{\alpha}$ enclose an area α under the normal distribution curve. Since the mean of the conditional distribution of Z_{t+l} is $\hat{z}_t(l) = E_t[Z_{t+l}]$ and by referring to a normal probability table it can be stated that the probability is 0.95 that the future value z_{t+l} will lie between the limits,

$$\hat{z}_t(l) \pm (1.96) \sigma(l)$$

These limits are said to be the 95% prediction limits for z_{t+l}.

Forecasts can be made for various lead times, but as the lead time increases, the accuracy of the forecasts obviously decreases.

2.3.6 Example

In this section a simple example of the modelling procedure is briefly outlined for a typical time series. The series, which is week-day peak load data for September 1991 to December 1991 (80 points), was plotted previously in Figure 2.3.3. (In the course of the modelling the series is scaled by a factor of 1.0E+3)

The first step in the modelling process is to identify a tentative model for the series. In Section 2.3.1 it was ascertained that this series requires one regular difference. The autocorrelation and partial autocorrelation functions for the differenced series are shown in Figures 2.3.5 and 2.3.7 respectively. Disregarding seasonal spikes at lags 5, 10, 15 etc. and associated 'satellite' spikes at lags 4, 6, 9, 11, etc. it can be construed that the sample ACF cuts off after lag 1 and the sample PACF tails off towards zero. Referring to the characteristic properties of regular operators, Table 2.3.1, this pattern merits the inclusion of a moving average operator of order 1 ($1-\theta_1 B$). Examination of the seasonal spikes at lags 5, 10, 15 etc. indicates that the sample autocorrelations at these seasonal lags appear to tail off towards zero and the sample partial autocorrelations cut off after lag 5. The characteristic properties of seasonal operators, Table 2.3.2, therefore identify a seasonal autoregressive operator of order 1 ($1-\phi_5 B^5$). The term, ($1-B$), represents the one regular difference applied. The resultant tentative model form is given below and the estimated parameters and the residual and model statistics listed.

$$(1-B)(1-\phi_5 B^5)y(t)=(1-\theta_1 B)e(t)$$

Differencing factors (order,degree): 1,1

		Factor	Lag	Coefficient	T-ratio
1	AR	1	5	0.83691	11.73
2	MA	1	1	0.13562	1.19

Residual and Model Statistics

Sum of Squares of the Residuals : 0.92161E-01

Mean Square of the Residuals: 0.11969E-02

R Squared Value : 0.95924

AIC : -6.7162

BIC : -6.6567

All of the coefficients are invertible. However, the moving average coefficient is not significant since its t-ratio does not lie outside the limits ± 1.96. So this parameter is deleted and a new tentative model estimated:

$$(1-B)(1-\phi_5 B^5)y(t)=e(t)$$

Differencing factors (order,degree) : 1,1

		Factor	Lag	Coefficient	T-ratio
1	AR	1	5	0.84929	12.45

Residual and Model Statistics

Sum of Squares of the Residuals : 0.93343E-01

Mean Square of the Residuals : 0.11967E-02

R Squared Value : 0.95871

AIC : -6.7285

BIC : -6.6987

The seasonal autoregressive term is invertible and the measures of model adequacy (sum of squares, R^2, AIC, BIC) have not changed significantly. Indicators of the sufficiency of this final model are given below.

Residual Analysis

Mean of the Residual Series : 0.22097E-03

Standard Deviation : 0.34158E-01

Mean Divided by the Standard Error of the Mean : 0.57861E-01

The residual autocorrelations are shown in Fig 2.3.8 and they ressemble white noise with the exception of randomly significant values.

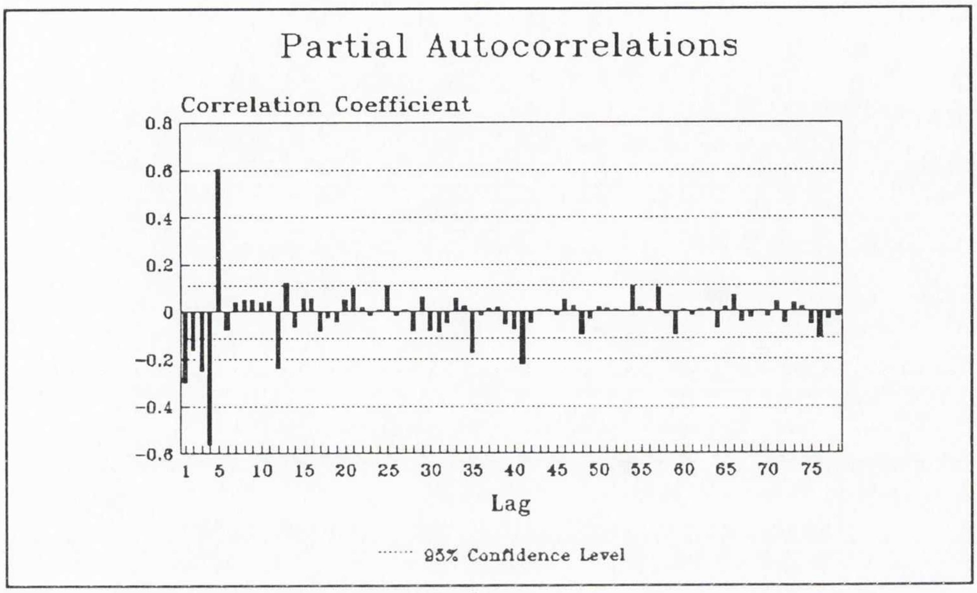

Figure 2.3.7 Partial Autocorrelations

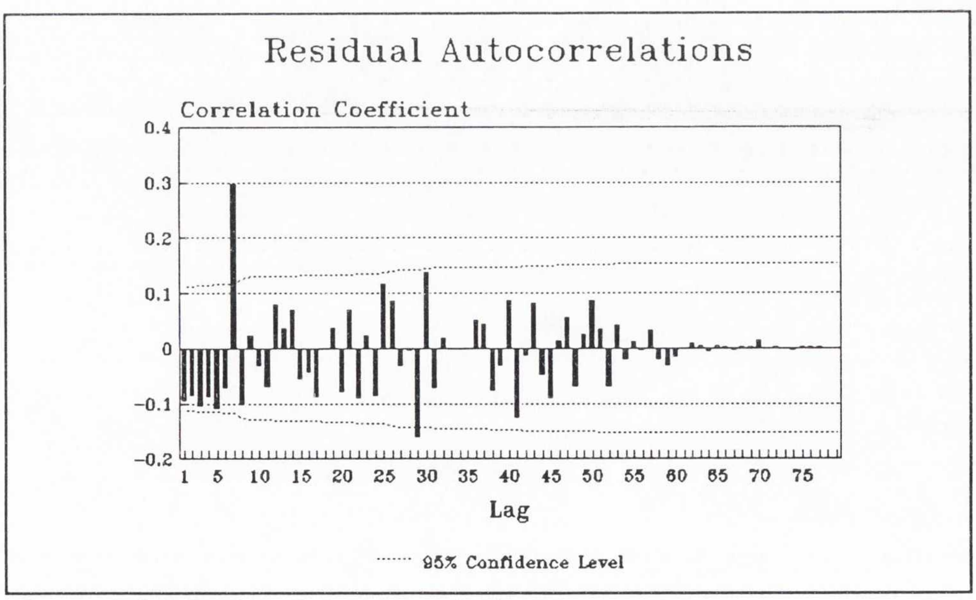

Figure 2.3.8 Residual Autocorrelations

The next step is to obtain forecasts using the estimated model. A one week ahead forecast (i.e. a forecast for the next 5 week days) was performed and Table 2.3.3 lists the actual and forecast values along with the 95% confidence limits. These findings are also plotted in Figure 2.3.9. Note that all the values must be multiplied by 1.0E+03 to determine the real predicted values.

Day	95% Lower Confidence Limits	Forecast	95% Upper Confidence Limits	Actual	% Error
Monday	1.3156	1.3856	1.4556	1.3750	0.77
Tuesday	1.2806	1.3796	1.4786	1.3760	0.26
Wednesday	1.2796	1.4008	1.5221	1.3930	0.56
Thursday	1.2209	1.3609	1.5009	1.3540	0.51
Friday	1.1246	1.2811	1.4376	1.2440	2.98

Table 2.3.3 Actual and Forecasted Week Day Peak Load (1 Week ahead)

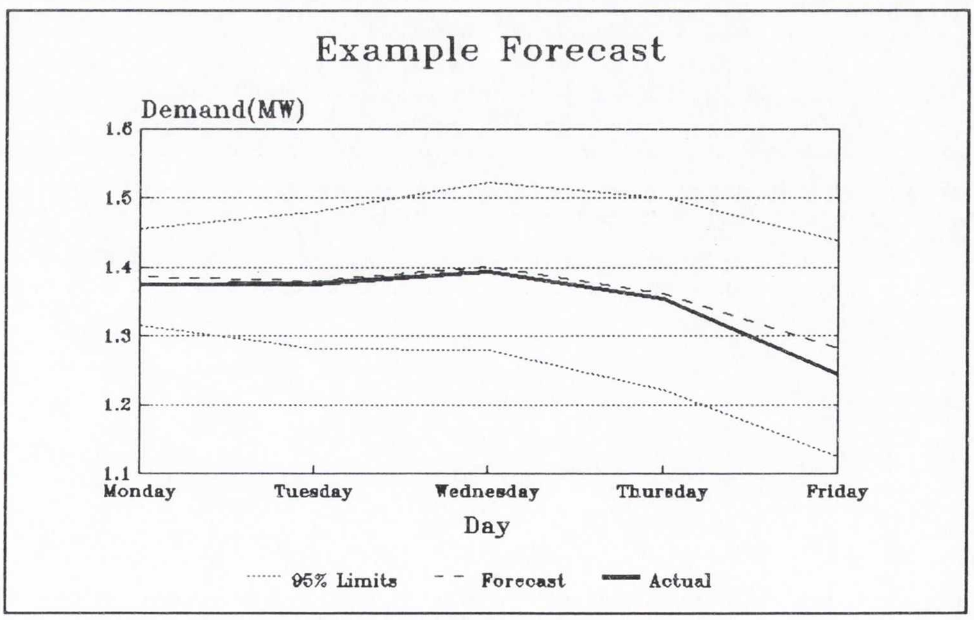

Figure 2.3.9 1 Week Ahead Peak Load Forecast

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3.1 Introduction

The procedures outlined in the previous chapter are now employed to develop univariate models for the prediction of total system electrical demand within the local utility. An economic evaluation of the univariate forecasts produced is also performed by ascertaining forecast effectiveness in relation to unit commitment decisions. The aim is to determine if unit start-up times are correctly determined even though the univariate demand predictions may be subject to errors.

3.2 Software Package

Initially two software packages, namely Statistical Package for the Social Sciences Extended version (SPSSx) and BioMeDical Programs (BMDP), were used in model development and forecasting. These were non-automatic packages with identification, estimation, diagnostic checking and forecasting having to be performed manually. However these packages were superseded by the main package employed in the model development - AutoBox Plus (ABP). This package has the advantage of using both an automatic or non-automatic modelling procedure. The automatic procedure follows the same steps as the non-automatic procedure except that the forecaster cannot interact with the procedure until it is finished. With the non-automatic, the skill and knowledge of the forecaster can be used with the modelling procedure to obtain the optimum models.

The first stage of the ABP development procedure is the identification of the tentative model and this again entails the use of the autocorrelation and partial autocorrelation functions. From the correlation plots it can be deduced if the time series is stationary - a prerequisite of Box-Jenkins modelling. If the series is non-stationary, then the differencing is applied to achieve a stationary series. The significant parameters of the tentative model can then be identified from the correlation plots.

The second phase of the model building process is the estimation of the coefficients in the tentatively identified model. This package uses a non-linear least squares estimation procedure that is based on the Marquardt algorithm (1). The program also makes use of the backcasting procedure discussed earlier. Diagnostic tests for necessity, invertibility and sufficiency are then performed. The test for necessity is performed by examining the t-ratios for the individual parameter estimates. Invertibility is determined by extracting the roots from each factor in the model to verify whether they lie outside the unit circle. Finally the patterns in the residual autocorrrelations and the partial autocorrelations are tested for white noise and parameters added to the model if necessary. If all the parameters are necessary, if each factor is invertible and if the model is sufficient, then the ARIMA model is adequate and it can be used for forecasting.

Model forecasting with the properly identified and estimated model is simply an algebraic process of applying the model form to the actual time series and computing the forecast values from a given time origin.

3.3 Demand Prediction

Models were developed using NIE hourly total system load data. Initially, week-days and week-ends were considered separately, but consideration of a continuous 7 day week proved a more viable option. Models were developed for Winter (Nov./Dec.), Spring (Mar./April), Summer (May/June) and Autumn (Sept./Oct.) using 6 weeks data, and the models were then used to forecast the following week. The model forms are outlined in the next section.

3.3.1 Model Development and Forecasting

The periodic nature of the load curve (i.e. the load at 10 a.m. on Tuesday is related to the load at 10 a.m. on Wednesday) is reflected by the use of a seasonal ARIMA $(p,d,q)\times(P,D,Q)_{24}$ model. It may be also necessary to recognise the weekly periodicity (i.e. Fridays are not like Saturdays), and a two period ARIMA $(p,d,q)\times(P,D,Q)_{24}\times(P^1,D^1,Q^1)_{168}$ model may be more applicable. An example of model development is now outlined.

Approximately six weeks of summer data (9th May - 23rd June 1991) were used to build the summer model. The first step in developing the model is to examine the autocorrelation function, Fig 3.3.1. This oscillatory non-decaying function indicates a non-stationary process. In order to obtain a stationary process seasonal differencing of order 24 and degree 1 (∇_{24}) was chosen. The autocorrelation function, Fig 3.3.2, and the partial autocorrelation function, Fig 3.3.3, of the resulting stationary process were then examined in order to obtain the model order. Since the autocorrelation function tails off towards zero and the partial autocorrelation function cuts off after lag 3 a regular AR(3) component is included in the model.

To facilitate the identification of the seasonal order (daily,weekly), multiples of 24 are pulled out of the ACF and PACF and inspected separately, Figures 3.3.4 and 3.3.5.

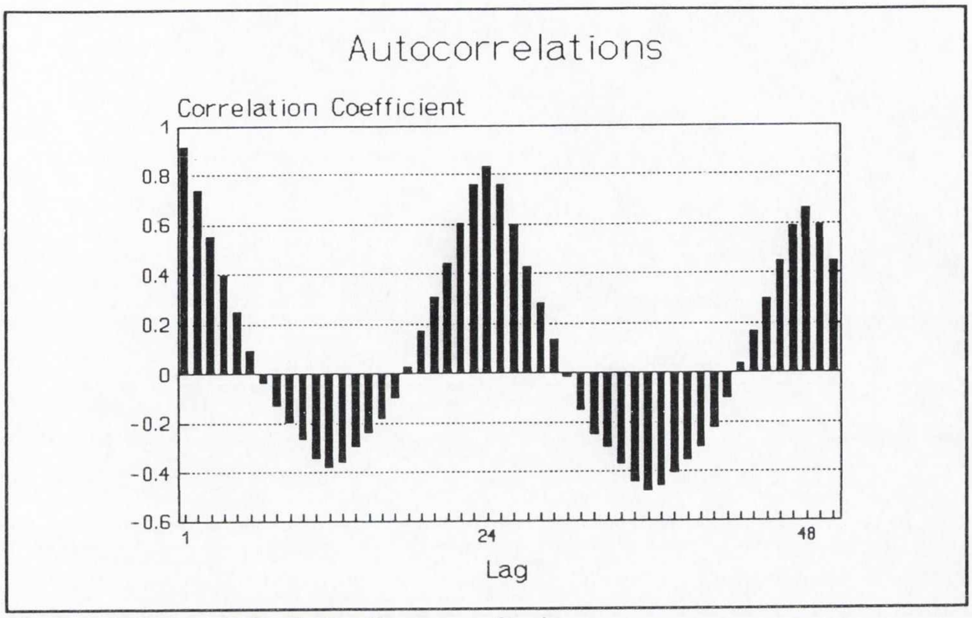


Figure 3.3.1 ACF of the Summer Series

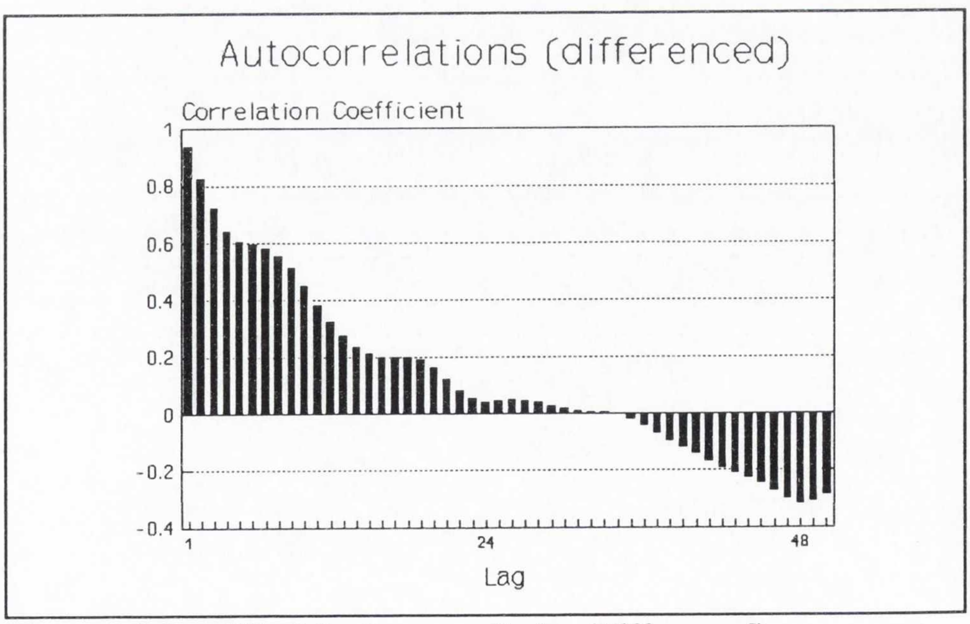


Figure 3.3.2 ACF of the Summer Series (Differenced)

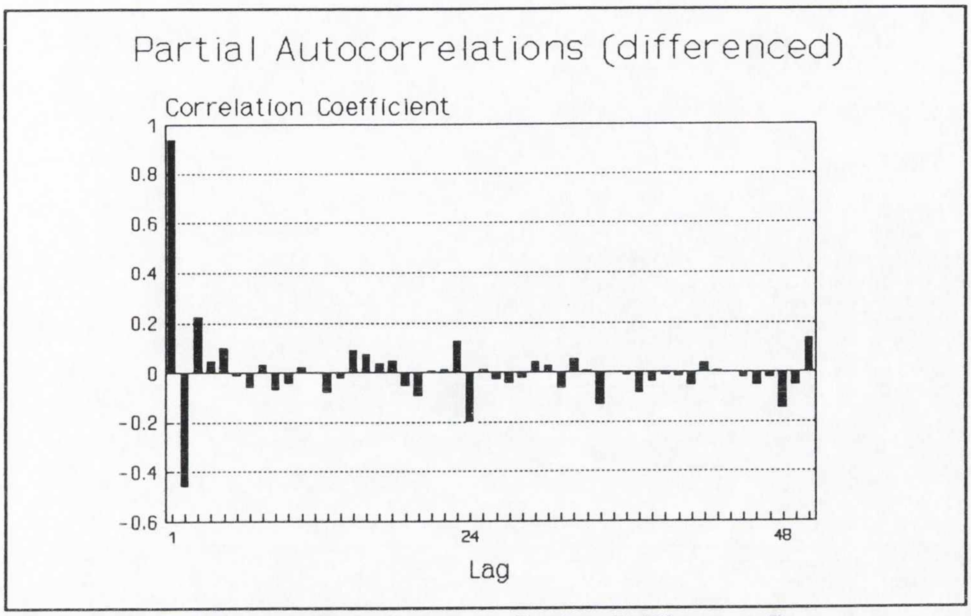


Figure 3.3.3 PACF of the Summer Series (Differenced)

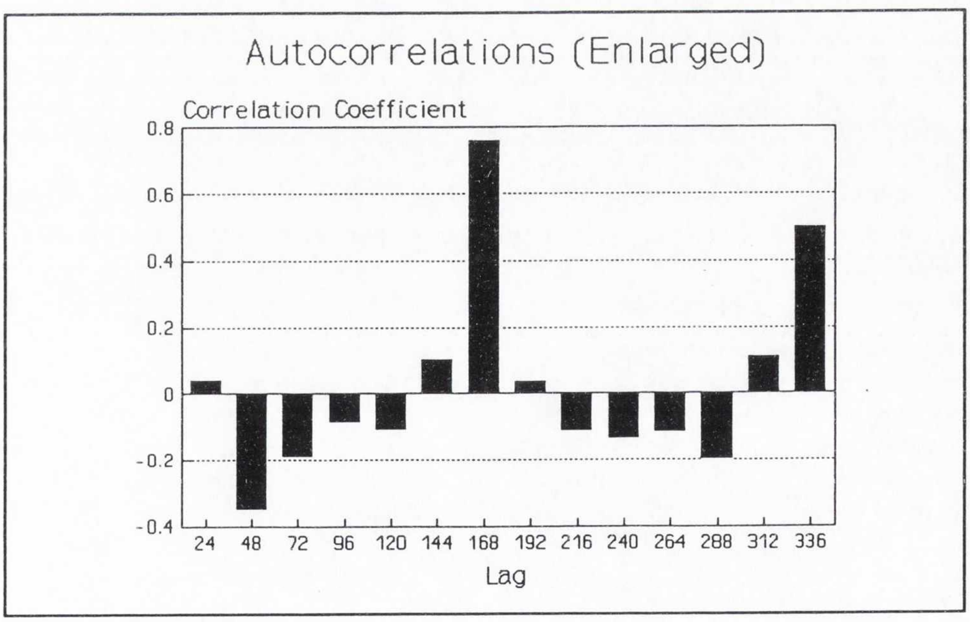


Figure 3.3.4 ACF Enlarged

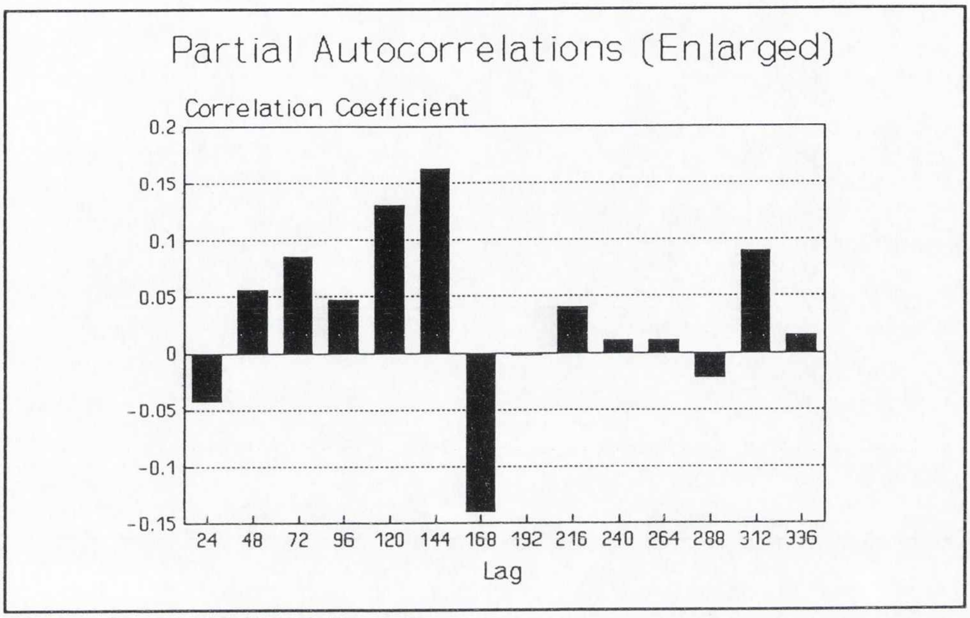


Figure 3.3.5 PACF Enlarged

From these figures it can be construed that the partial autocorrelations at lags 24, 48, 96, ... tail off towards zero and the seasonal autocorrelations cut off after lag 48. This scenario suggests the inclusion of a second order seasonal moving average term. The plots also indicate that the autocorrelations at weekly lags decay slowly and that the partial autocorrelations cut off after lag 168. This warrants the addition of a weekly AR(1) factor. Hence our tentative model is of the form $(3,0,0)\times(0,1,2)_{24}\times(1,0,0)_{168}$. The tentative model form is given below and the parameter estimates and residual and model statistics listed.

$$(1-B^{24})(1-\phi_1 B-\phi_2 B^2-\phi_3 B^3)(1-\phi_{168}B^{168}) = (1-\theta_{24}B^{24}-\theta_{48}B^{48})e(t)$$

Differencing Factors (Order,Degree): 24,1

		Factor	Lag	Coefficient	T-Ratio
1	AR	1	1	0.93912	27.58
2	AR	1	2	-0.037663	-0.84
3	AR	1	3	-0.013812	-0.45
4	AR	2	168	0.90505	59.42
5	MA	1	24	0.69489	20.39
6	MA	1	48	0.07902	2.27

Residual and Model Statistics

Sum of Squares of the Residuals : 0.11302

Mean Square of the Residuals : 0.12857E-03

R Squared Value : 0.99462

AIC : -8.9523

BIC : -8.9198

Following the iterative process of tentative model identification, estimation and diagnostic checking, a final tentative model was determined, details of which are outlined. Measures of sufficiency for this final model are also given and the residual autocorrelations are shown in Figures 3.3.6 and 3.3.7.

Differencing Factors (Order,Degree) : 24,1

		Factor	Lag	Coefficient	T-Ratio
1	AR	1	1	0.89328	59.06
2	AR	2	168	0.91978	65.07
3	MA	1	24	0.70449	27.91

Residual and Model Statistics

Sum of Squares of the Residuals : 0.11387

Mean Square of the Residuals : 0.12881E-03

R Squared Value : 0.99459

AIC : -8.9538

BIC : -8.9376

Diagnostic Checks for Sufficiency

Mean of the Residual Series : -0.46972E-04

Standard Deviation : 0.01133

Mean Divided by the Standard Error of the Mean : -0.12347

These tests for model sufficiency and fit give no strong indication of model inadequacy. Other model forms were also considered but this was chosen to be the optimum.

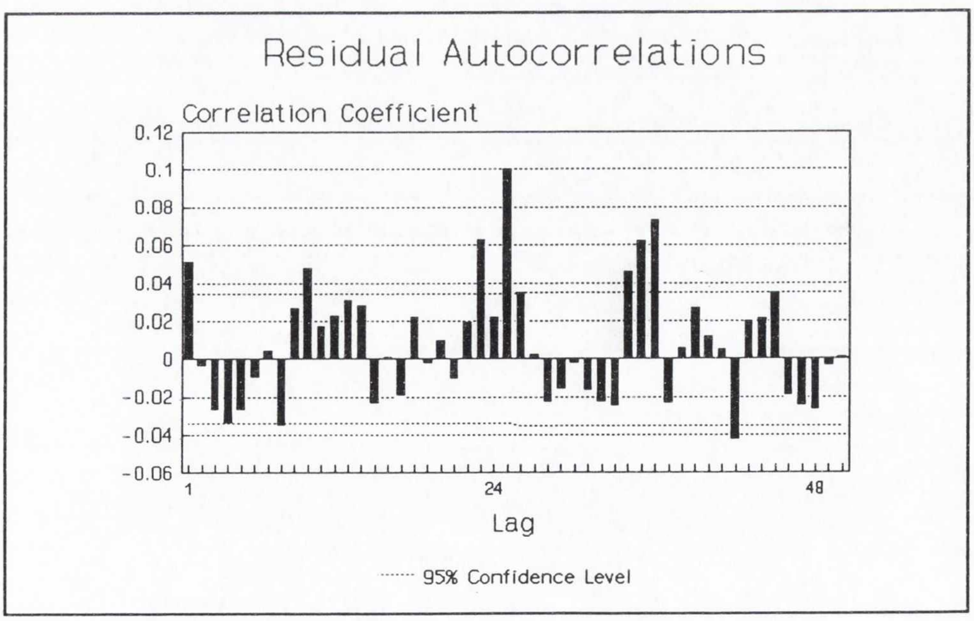


Figure 3.3.6 Residual ACF

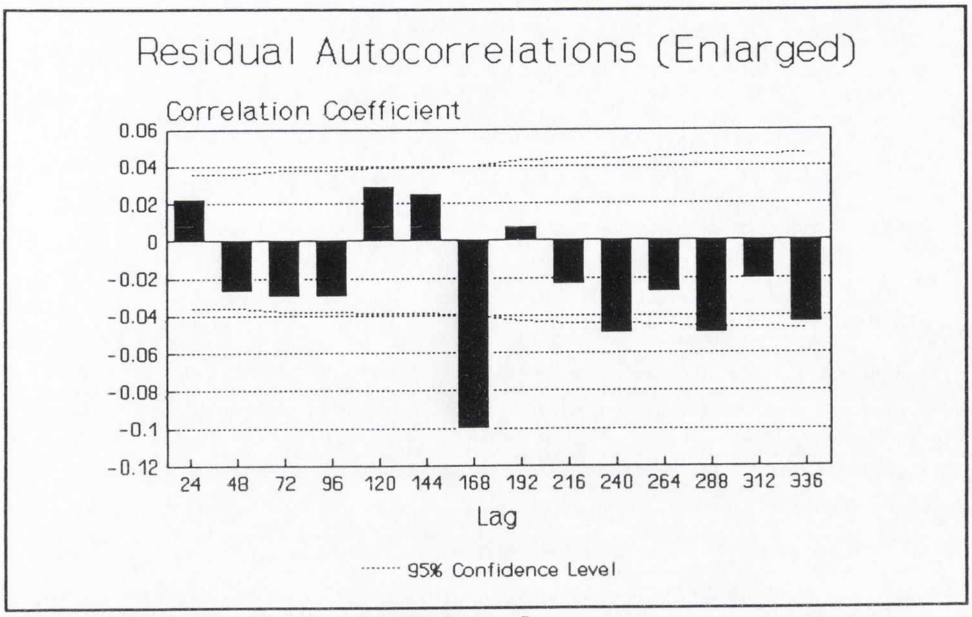


Figure 3.3.7 Residual ACF Enlarged

Similar steps were employed to develop models for winter, spring and autumn. The model forms for all four seasons are now given,

Winter : $(1-B^{24})(1-\phi_1B-\phi_2B^2)(1-\phi_{168}B^{168}) = (1-\theta_{24}B^{24}-\theta_{48}B^{48})e(t)$

Spring : $(1-B^{24})(1-\phi_1B-\phi_2B^2-\phi_3B^3)(1-\phi_{168}B^{168}) = (1-\theta_{24}B^{24}-\theta_{48}B^{48})e(t)$

Summer : $(1-B^{24})(1-\phi_1B)(1-\phi_{168}B^{168}) = (1-\theta_{24}B^{24})e(t)$

Autumn : $(1-B^{24})(1-\phi_1B-\phi_2B^2-\phi_3B^3)(1-\phi_{168}B^{168}) = (1-\theta_{24}B^{24})e(t)$

The parameter values are listed in Table 3.1 and Fig 3.3.8 highlights the parameter changes between seasons. Although the coefficients of the parameters vary widely between seasons, there is no radical parameter variation. Indeed, the models for each season follow the same basic form. Winter and summer load is mainly shaped by the previous hour's load, the load at the same time yesterday and the load at the same time on the same day of the previous week. However, in spring and autumn dependency on load at other lags is more marked.

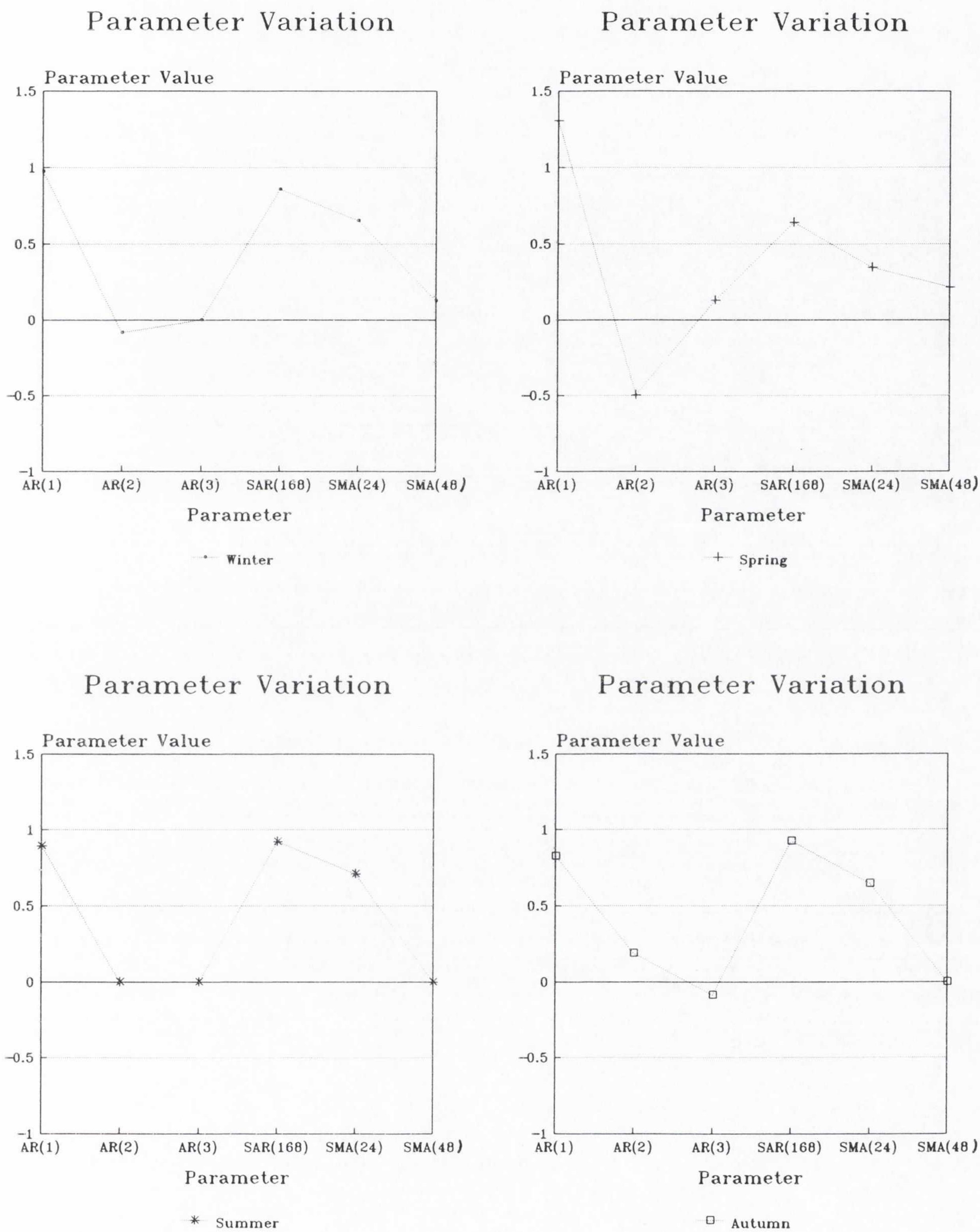


Figure 3.3.8 Parameter Variation For Each Season

Parameter	Winter	Spring	Summer	Autumn
ϕ_1	0.9727	1.3088	0.8939	0.8252
ϕ_2	-0.0826	-0.4973	0.0	0.1415
ϕ_3	0.0	0.1318	0.0	-0.0857
ϕ_{168}	0.8564	0.6451	0.9194	0.9230
θ_{24}	0.6536	0.3474	0.7083	0.6452
θ_{48}	0.1290	0.2173	0.0	0.0

Table 3.1 Parameter Values

3.3.2 Demand Forecasts

The usual way to test the ability of models is to model using data up to a given date and then to analyse the forecasts after that date. So, after using six weeks data for model development, the following week was used to check the performance of the models.

Initially 24h ahead forecasts were produced for each day of the following week. Figures 3.3.9 to 3.3.12 show the actual and predicted hourly load for each week day (beginning with Monday) in each of the four seasons. A simple measure of accuracy is provided by the mean absolute percentage error (MAPE). The term is self-explanatory and is defined as:

$$MAPE=\frac{100}{n}\sum_{i=1}^{n}\frac{|\bar{y}_i-y_i|}{y_i}$$

where $\hat{y}_i$ is the predicted value, and y_i is the actual value. The actual and percentage errors are shown in Figures 3.3.13 to 3.3.16.

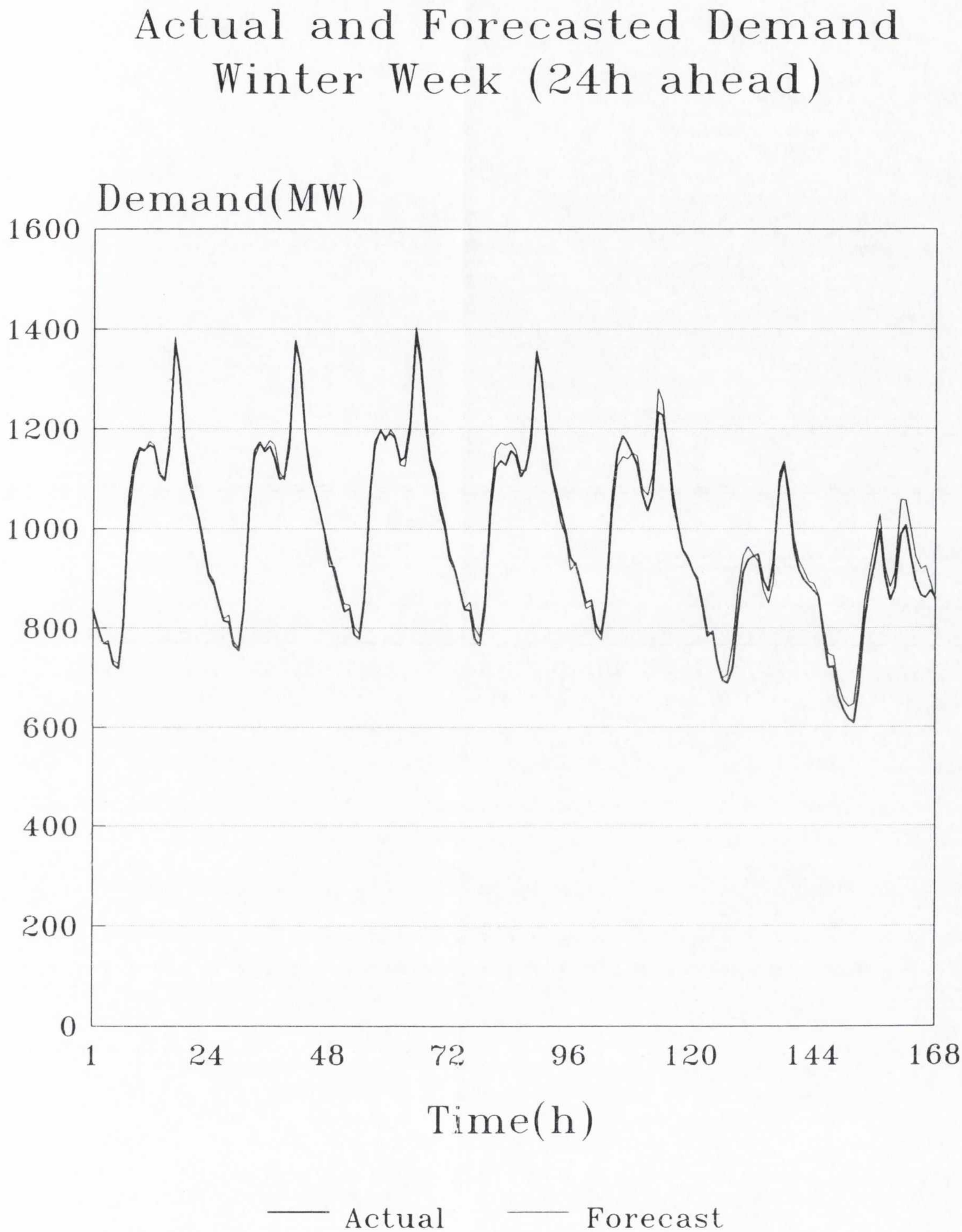


Figure 3.3.9 Actual and Forecasted Demand - Winter Week (24h ahead)

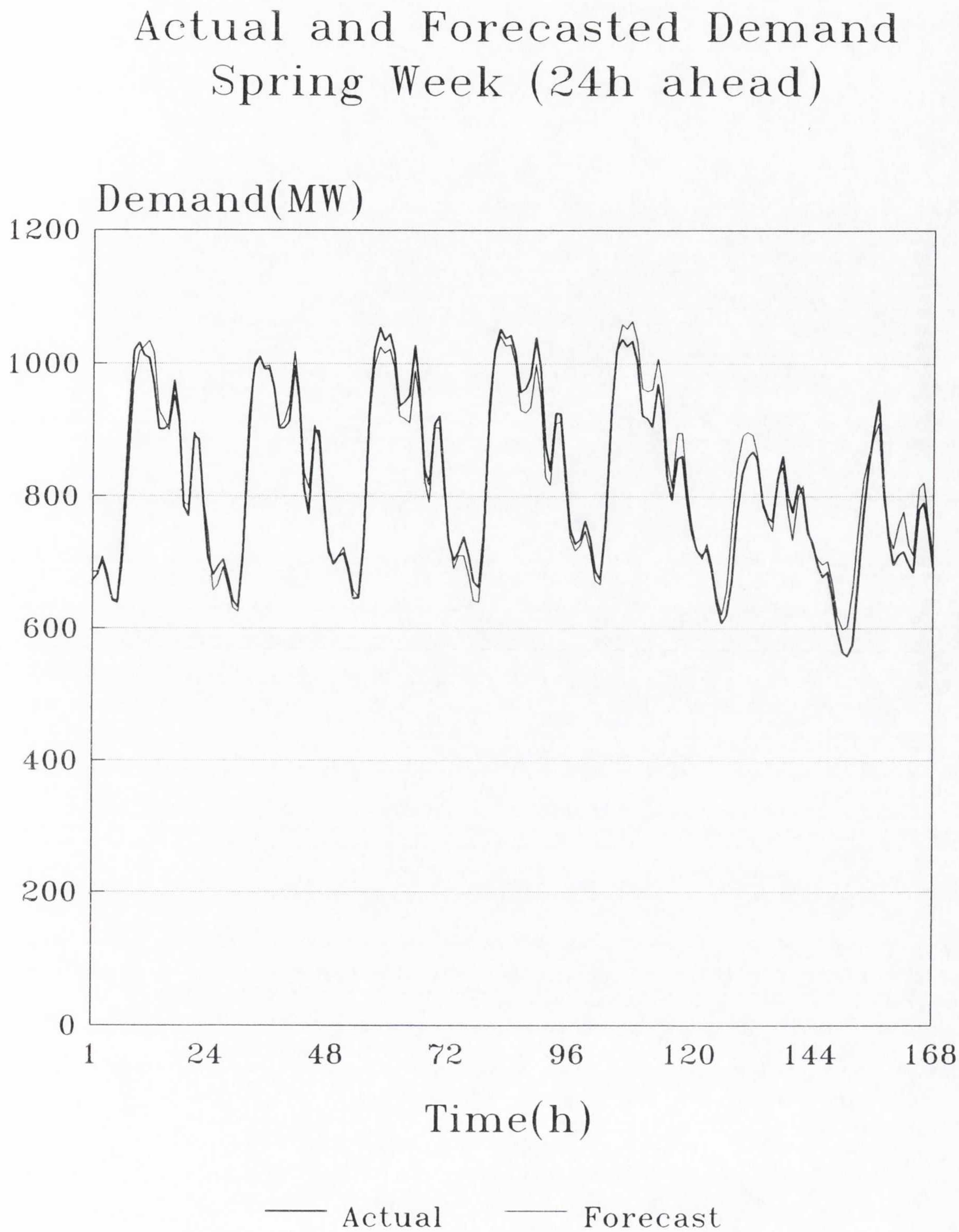


Figure 3.3.10 Actual and Forecasted Demand - Spring Week (24h ahead)

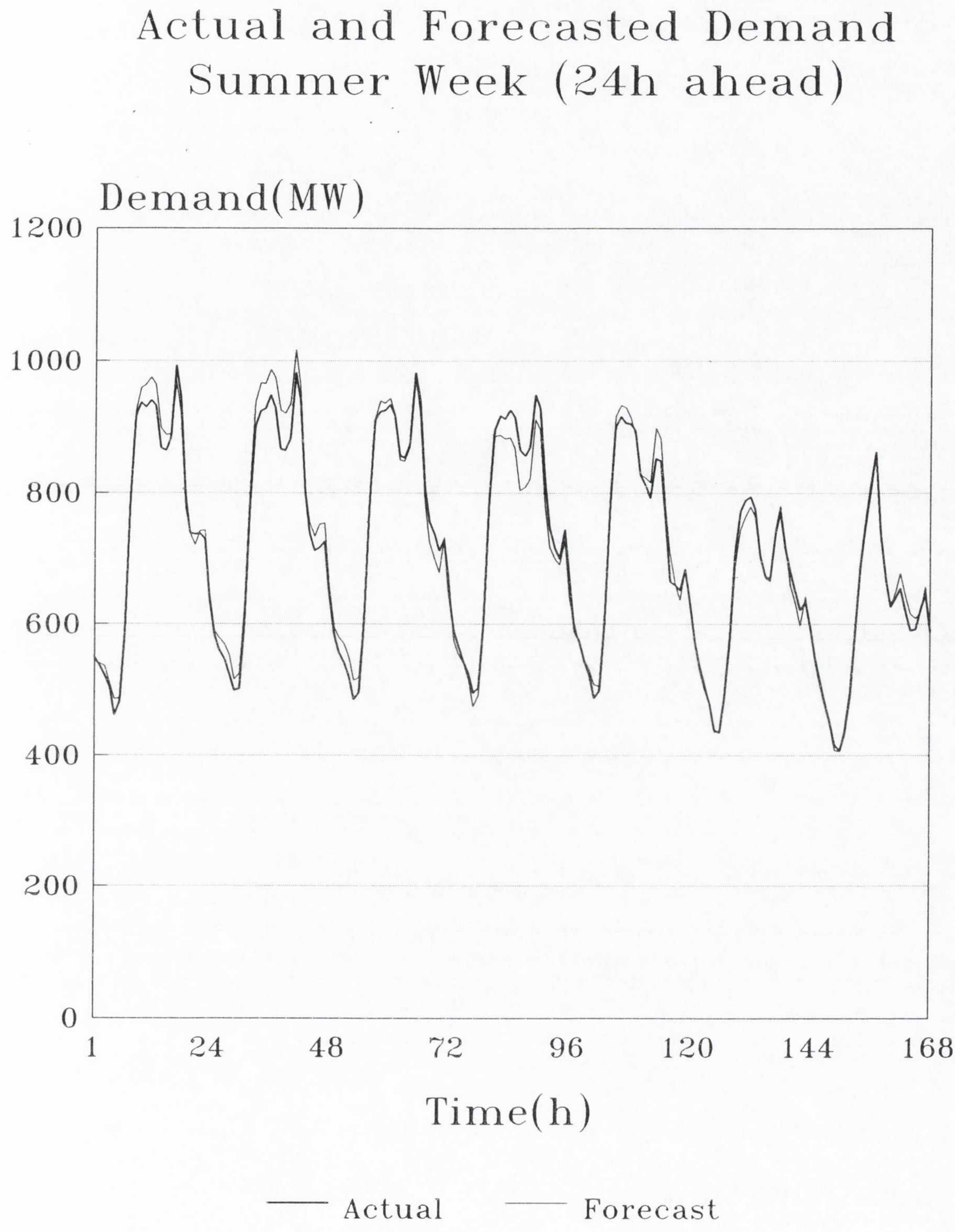


Figure 3.3.11 Actual and Forecasted Demand - Summer Week (24h ahead)

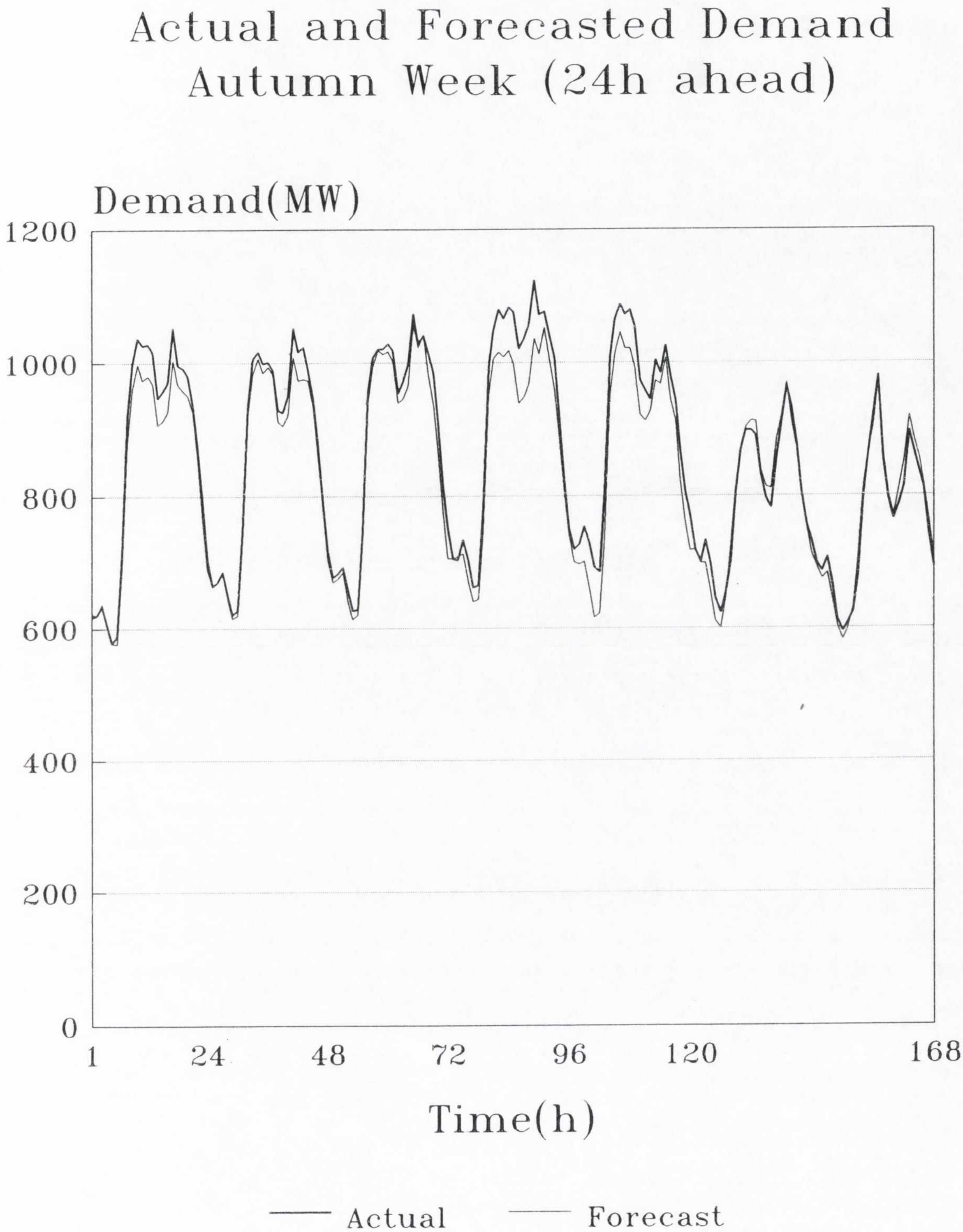
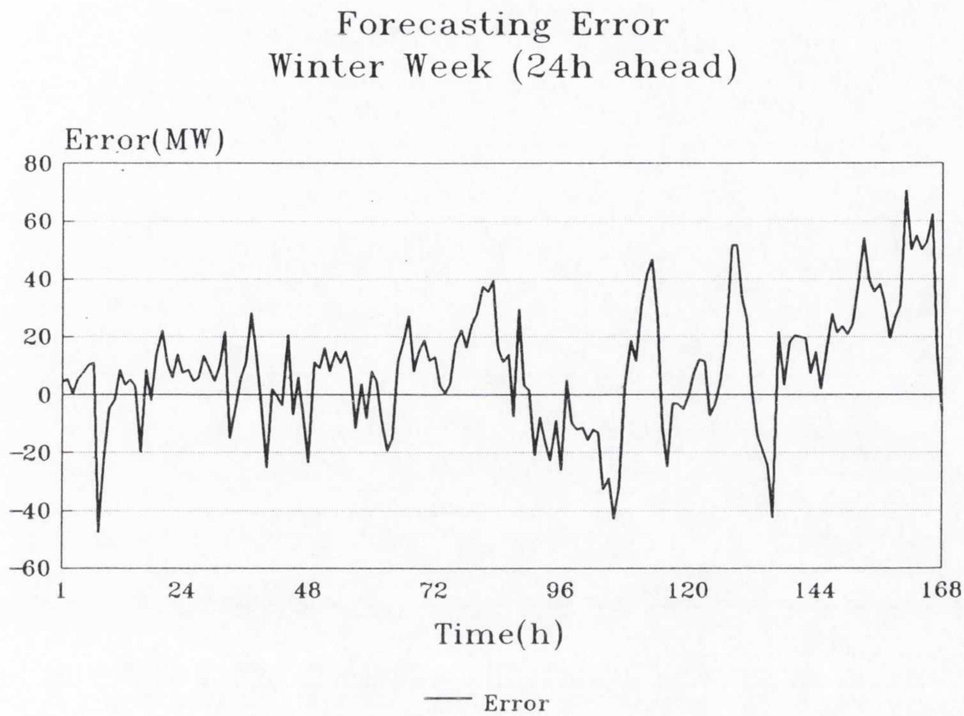


Figure 3.3.12 Actual and Forecasted Demand - Autumn Week (24h ahead)



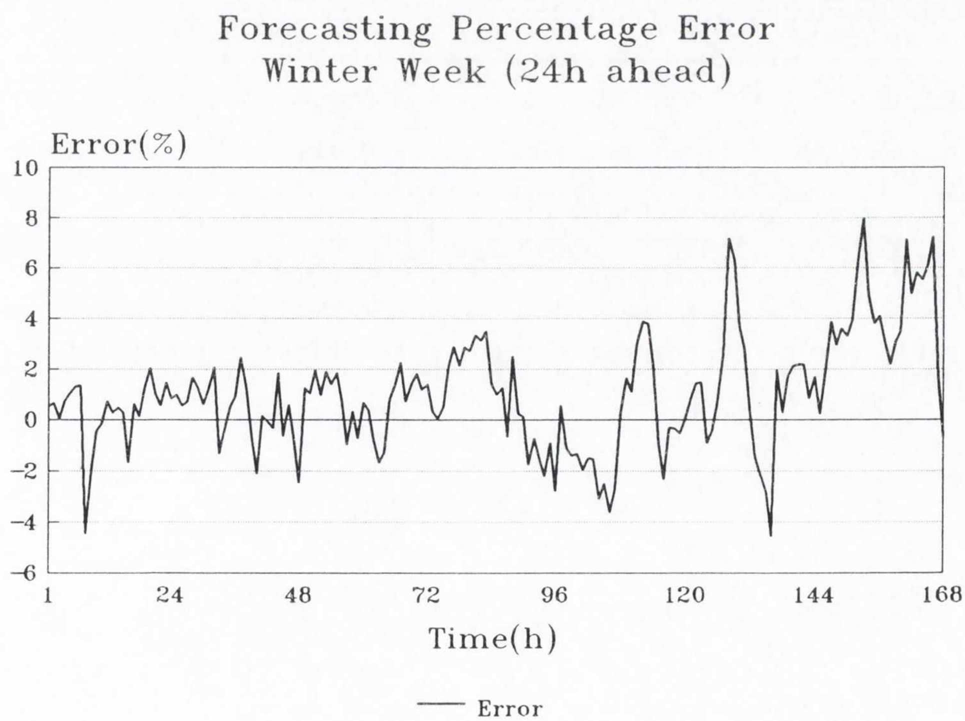
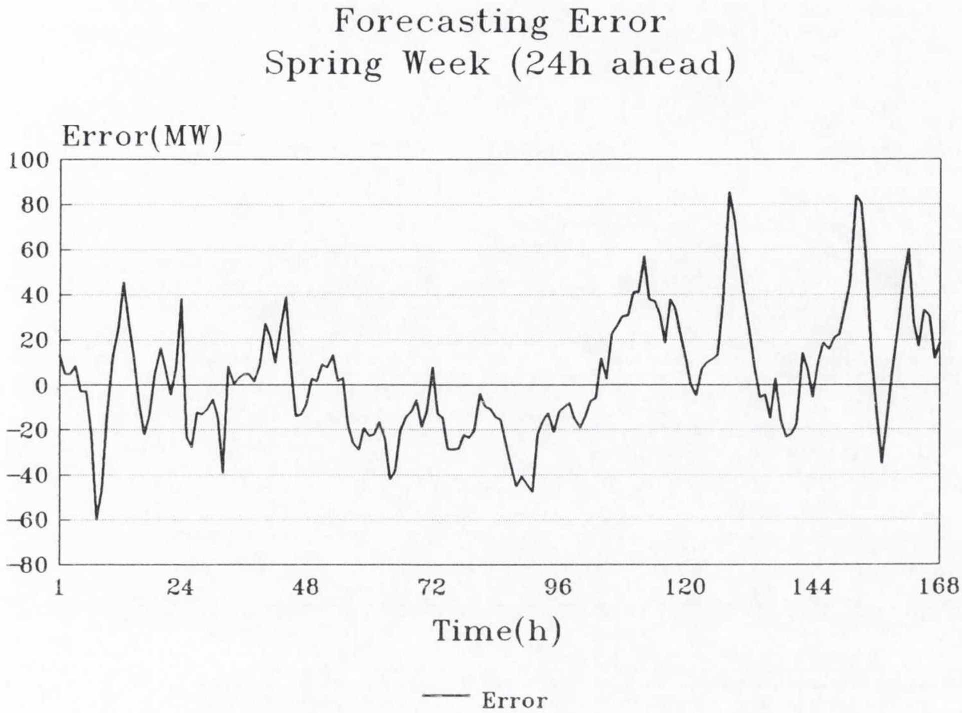


Figure 3.3.13 Actual and Percentage Errors - Winter Week (24h ahead)



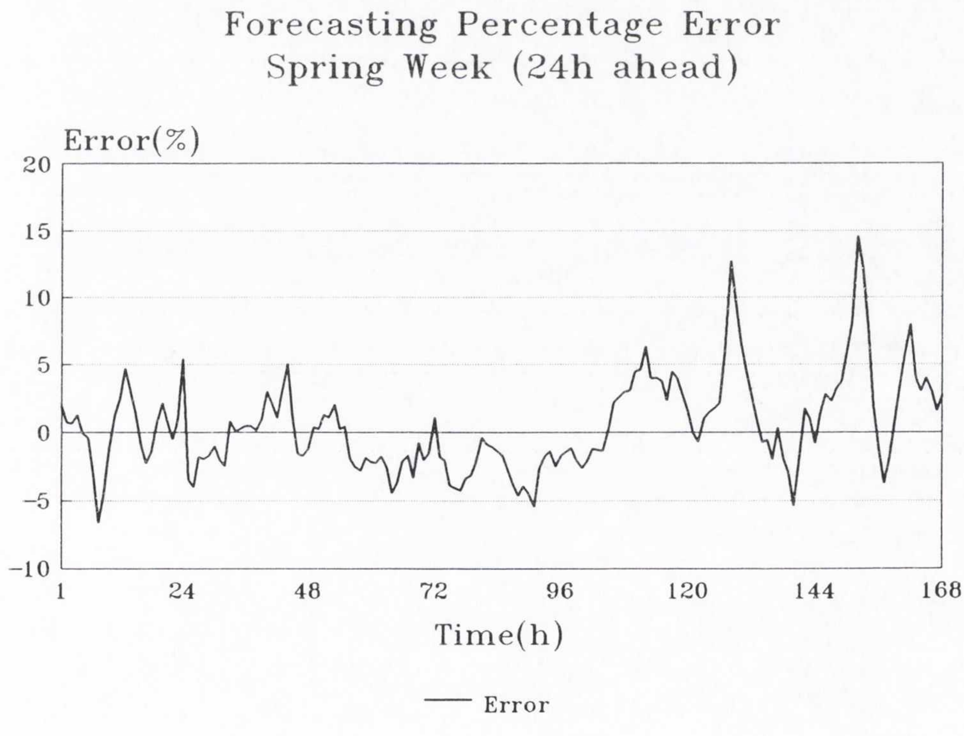
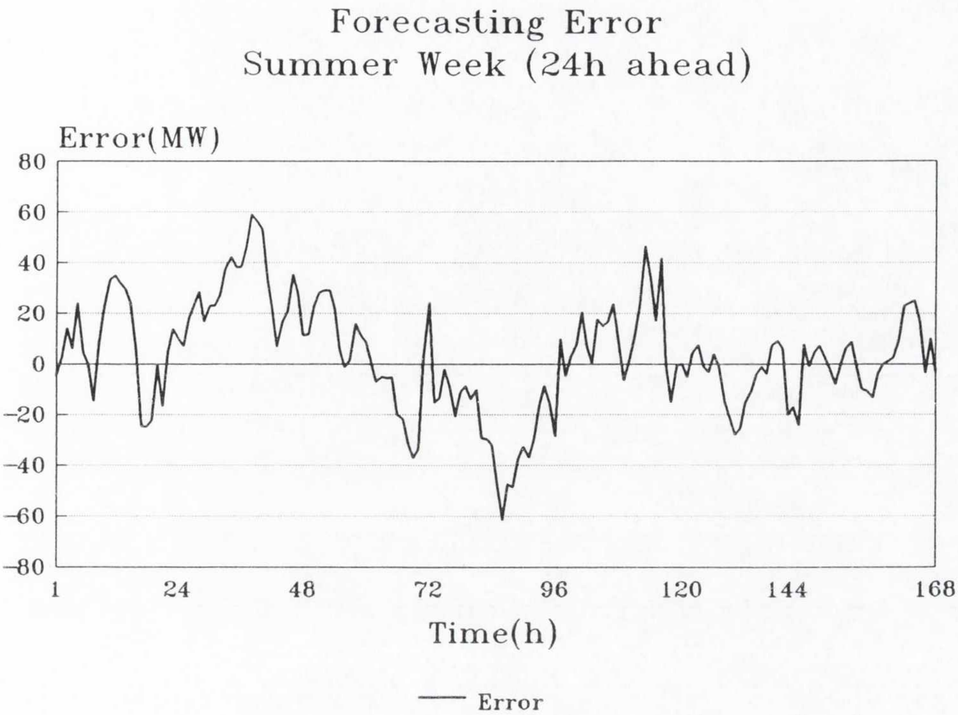


Figure 3.3.14 Actual and Percentage Errors - Spring Week (24h ahead)



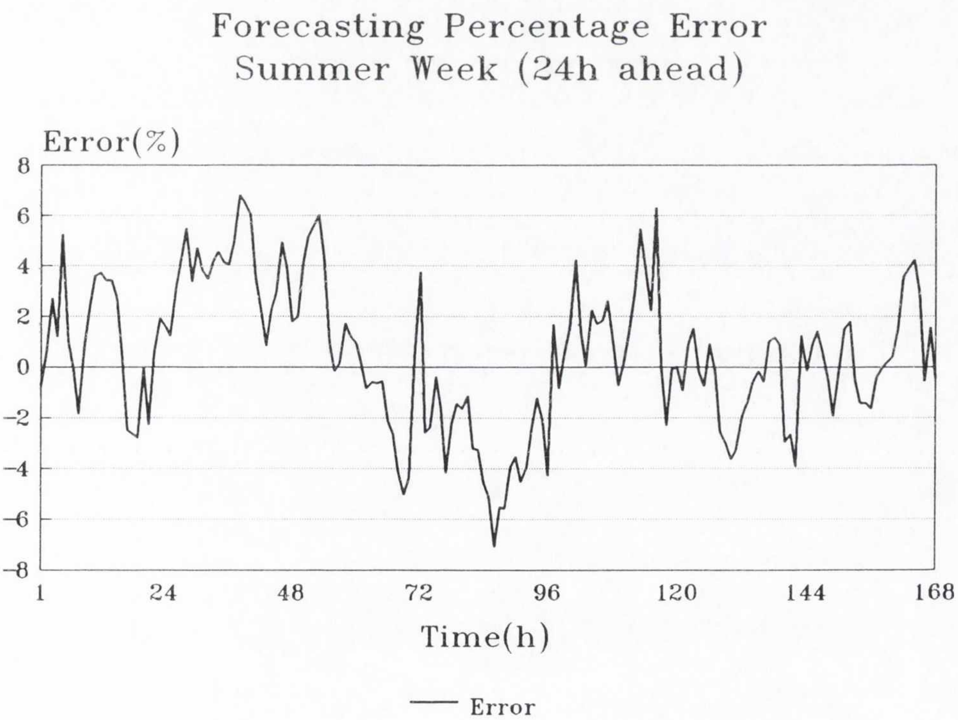
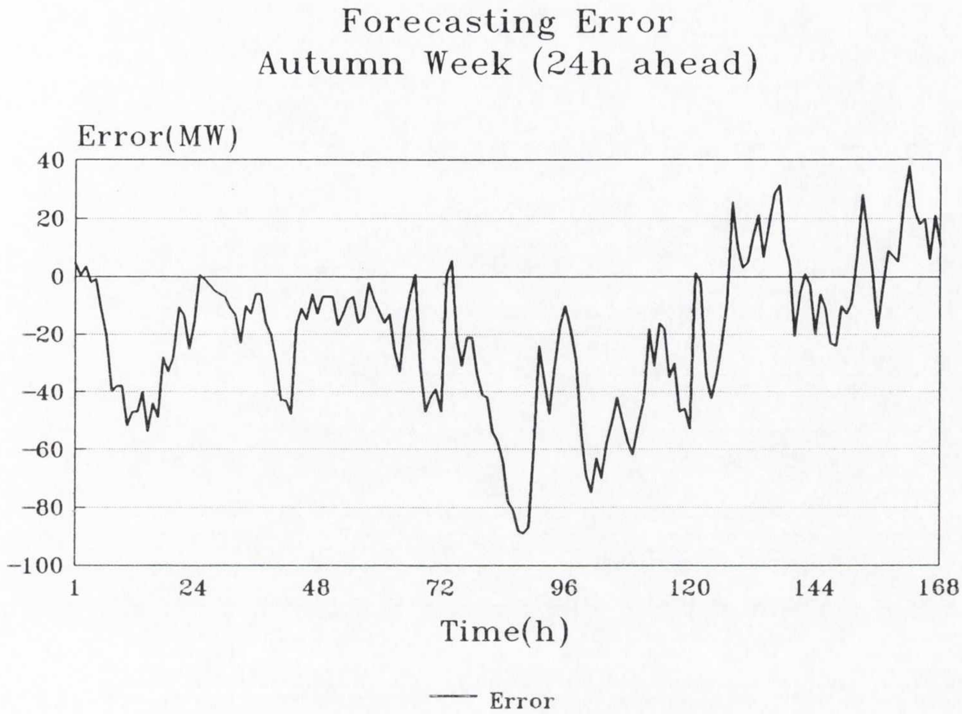


Figure 3.3.15 Actual and Percentage Errors - Summer Week (24h ahead)



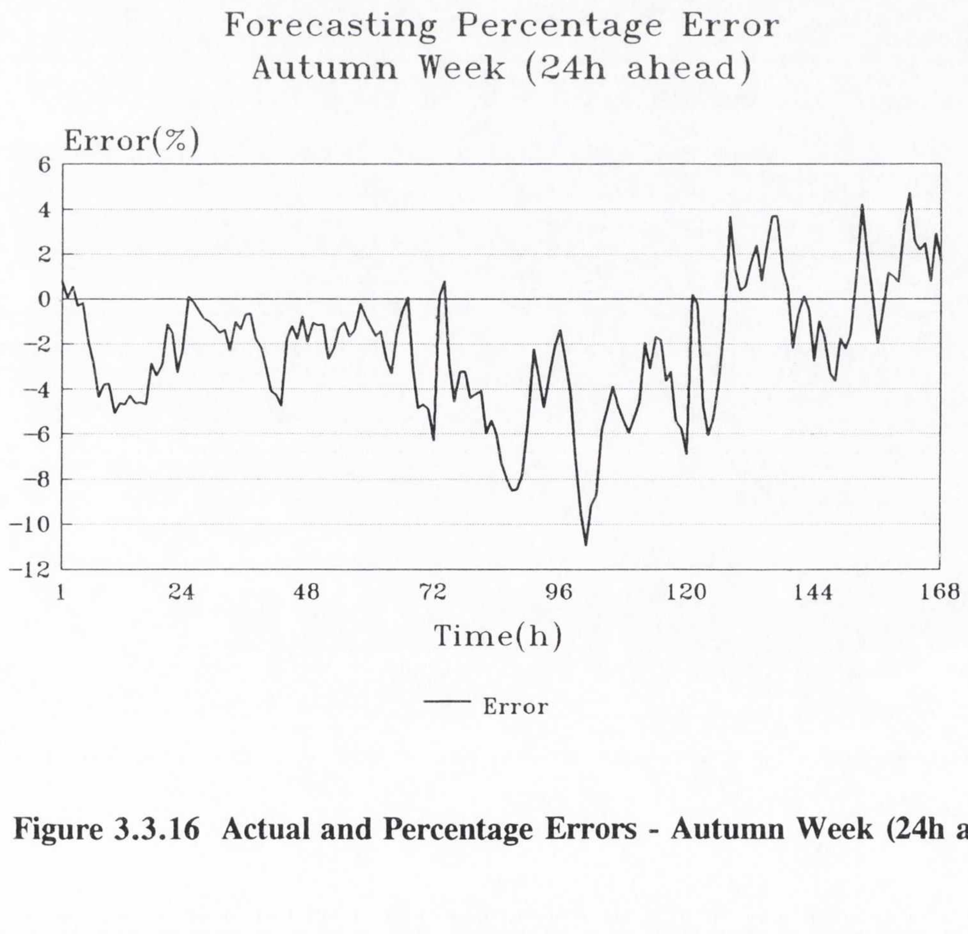


Figure 3.3.16 Actual and Percentage Errors - Autumn Week (24h ahead)

Since the Box-Jenkins predictor extrapolates trends of increasing or decreasing demand level it often exaggerates perturbations in the recent demand history. The errors also exhibit seasonality with the largest errors occurring in spring and autumn. During the spring and autumn customers are sensitive to weather changes, continually switching heating appliances and lights on and off. This causes a large change in the demand in response to temperature/weather changes. In the winter most central heating systems are automatically controlled and electric heaters are often switched on simply because it is winter, and the demand sensitivity to temperature/weather decreases from autumn and spring peaks. In summer the presence of frontal systems bringing alternately wet, dull weather and dry, bright weather upsets forecast accuracy. Generally, during all seasons, if a change in weather conditions occurs, then the corresponding prediction error is large.

A qualitative insight into the influence of weather phenomema may be gained by considering the predictions obtained for the Thursday and Friday in Autumn. Figures 3.3.12 and 3.3.16 indicate that the predictions fall well below the actual demand, i.e there may have been an abnormal increase in demand. A cursory examination of corresponding weather data shows that temperature falls from 12 °C at the beginning of the week to approximately 5 °C on Thursday/Friday with the windchill equivalent temperature also plummeting from 10 °C to -3 °C. So the consideration of weather factors in the prediction process may improve forecast accuracy. A further investigation of the relationship between demand and meteorological factors will be carried out in Chapter Six.

3.3.3 Forecast Lead Time

Forecasts of future demand can be made using different lead times, with the optimum forecast most likely coming from the shortest lead time. As shown in Section 2.3.5, if forecasts for more than one step ahead are required, then it is necessary to use previous forecasts to achieve predictions of future demand. Hence the use of previous forecasts decreases the probability of the prediction and increases the confidence limits.

Previous forecasts have concentrated on 24h ahead predictions, with the result that the influence of weather variables at this relatively large lead time can produce added errors. The minimum forecast lead time here is one hour and Figures 3.17 to 3.20 show the predictions for 1h ahead and 24h ahead for the most inaccurate (in terms of MAPE) winter, spring, summer and autumn day predictions. A comparison of the MAPE errors for these different lead times is given in Table 3.2. The MAPE errors are considerably improved by forecasting at this minimum or optimum lead time. This may be due to the fact that weather effects do not cause significant errors at this smaller lead time because weather phenomena require a relatively longer time to develop.

Lead Time	Winter MAPE(%)	Spring MAPE(%)	Summer MAPE(%)	Autumn MAPE(%)
24h	3.58	3.85	3.64	5.25
1h	1.41	1.92	1.20	0.98

Table 3.2 24h ahead and 1h ahead MAPE

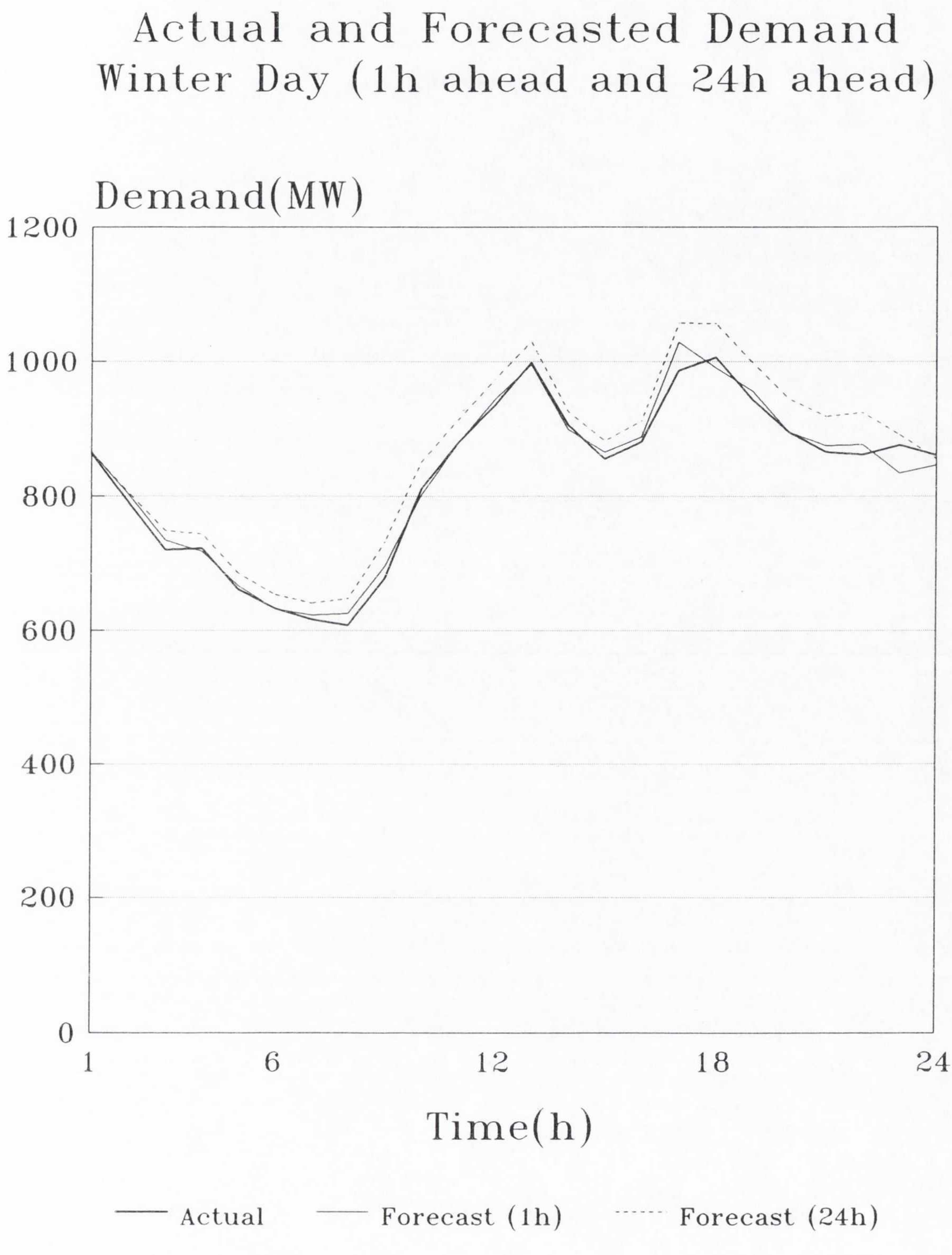


Figure 3.3.17 Actual and Forecasted Demand - Winter Day

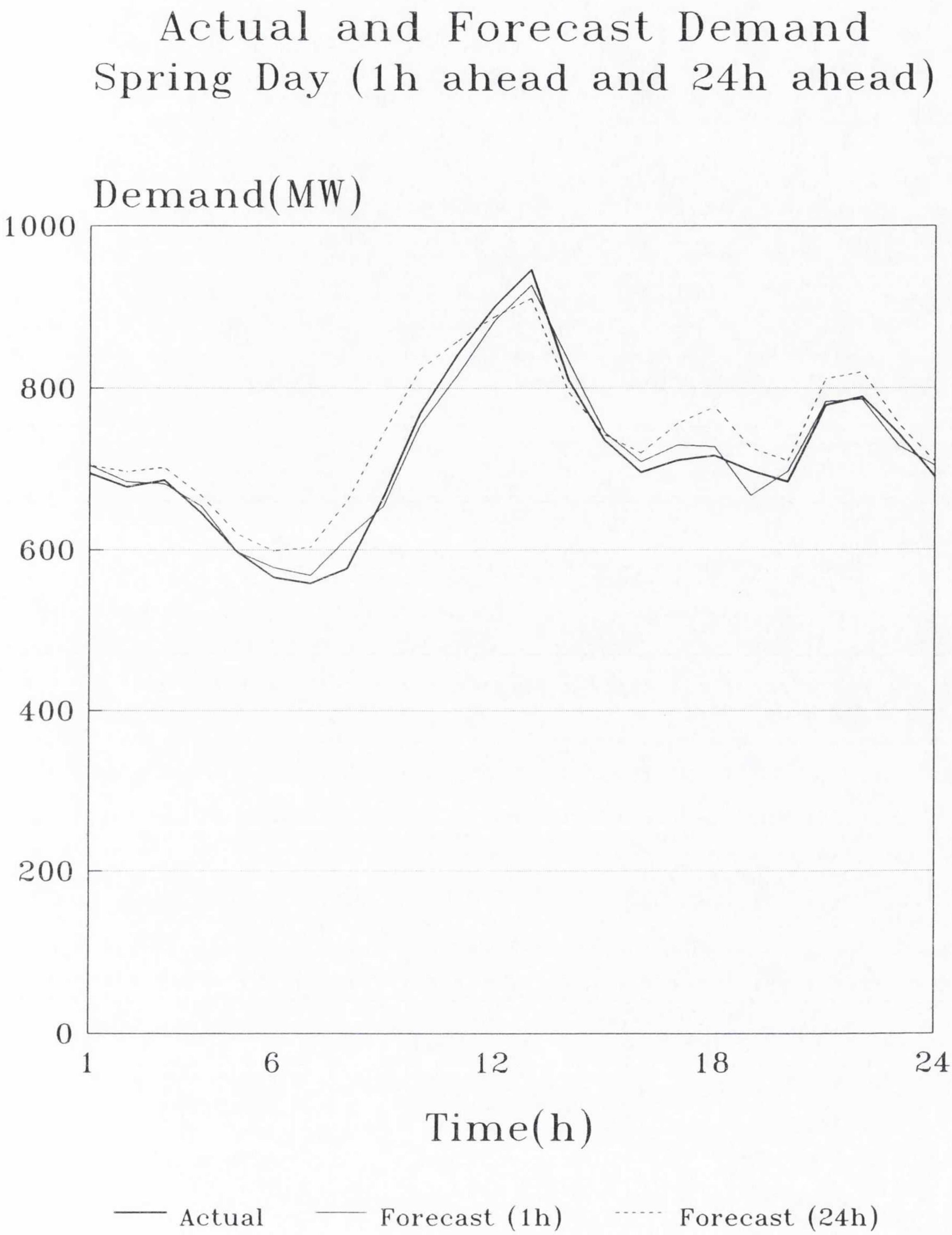


Figure 3.3.18 Actual and Forecasted Demand - Spring Day

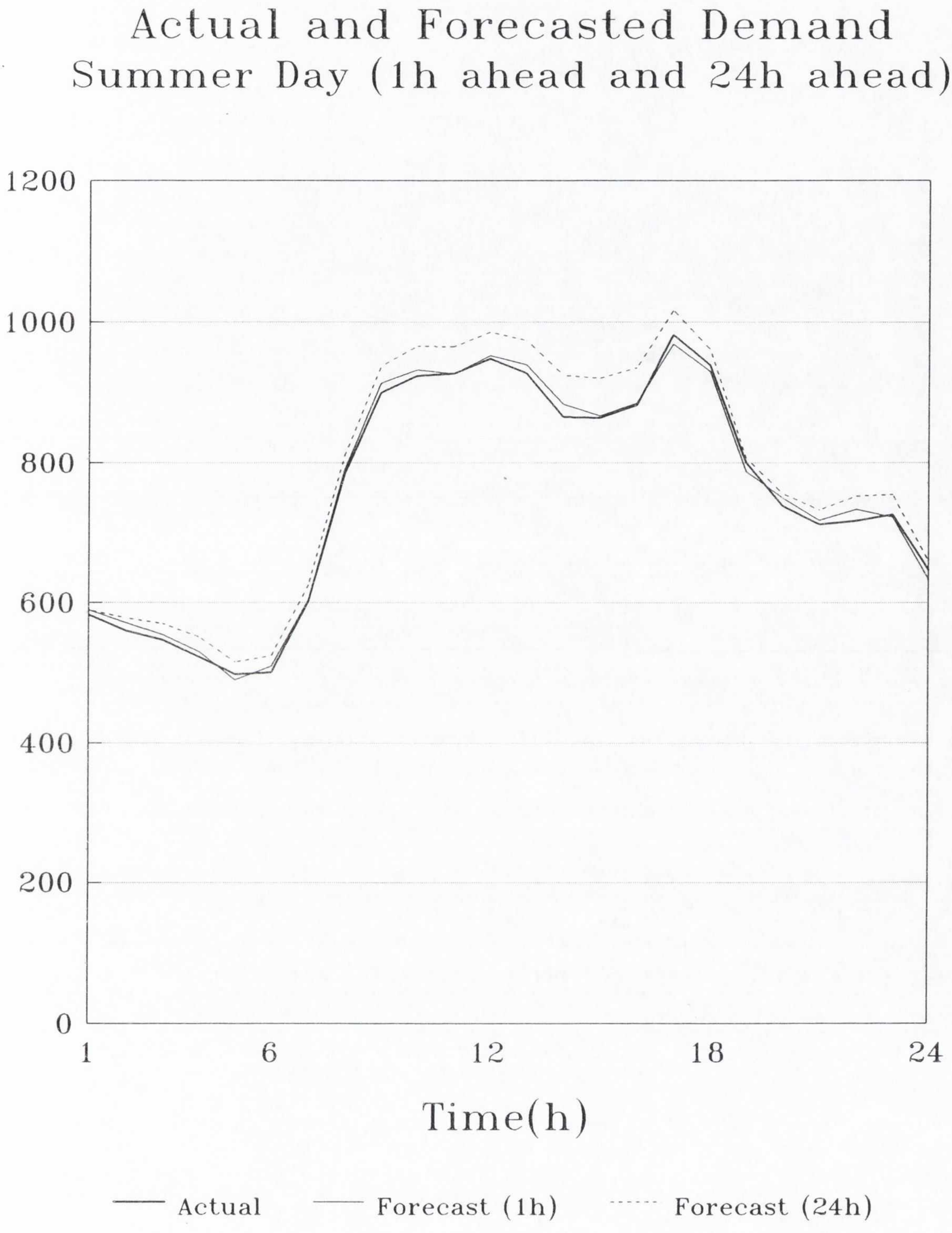


Figure 3.3.19 Actual and forecasted Demand - Summer Day

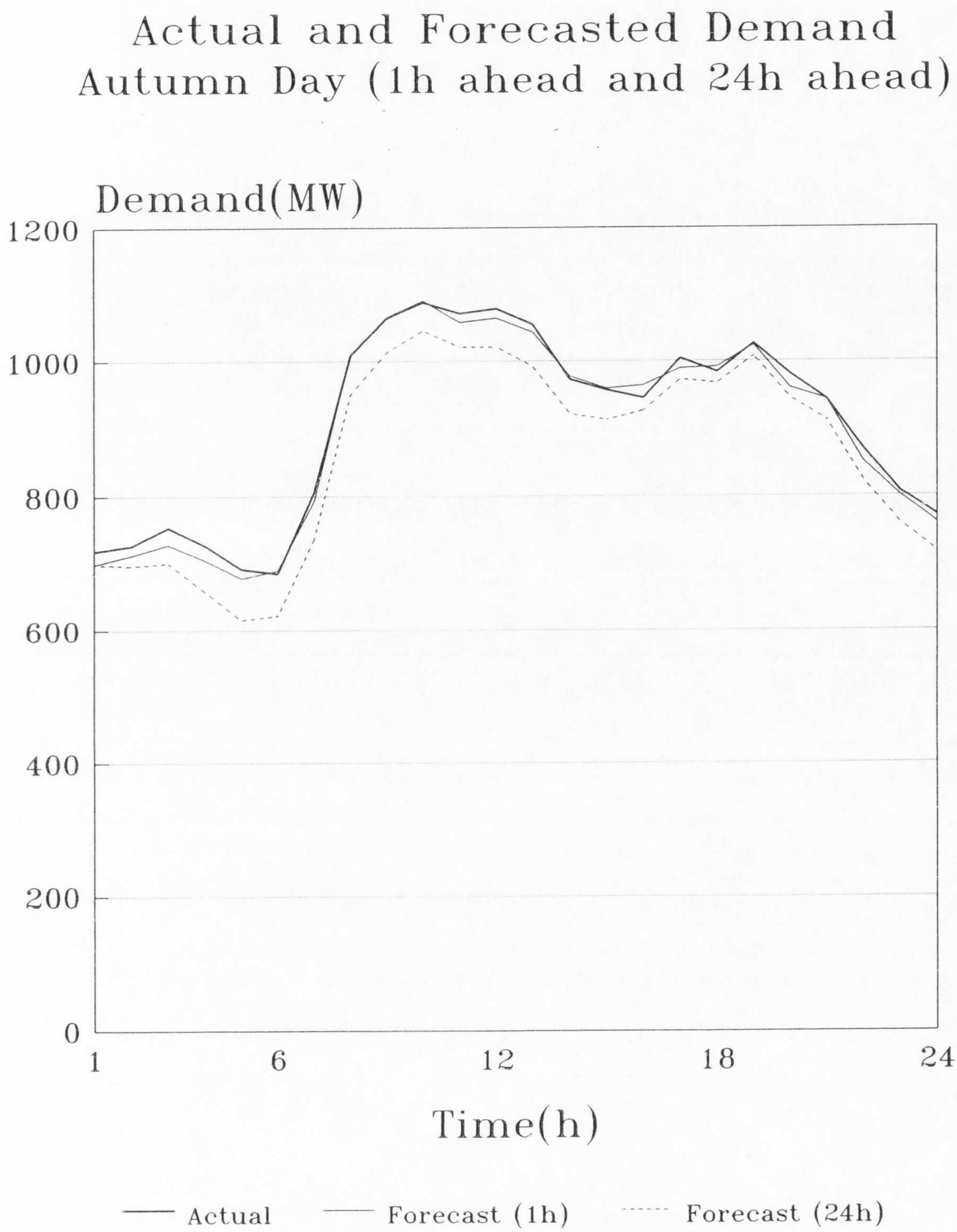


Figure 3.3.20 Actual and Forecasted Demand - Autumn Day

3.4 Forecasting Deficit

3.4.1 Introduction

The economic consequences of not forecasting demand were evaluated in Chapter One and it was shown that failure to forecast leads to a 2%-3% increase in operating costs. So having obtained univariate predictions, the next step is to determine the increase in operating cost or forecast deficit caused by forecast inaccuracy.

On-line system operation has the function of deciding which generator sets should be on-line to provide a predicted demand and reserve (2). This operation may be summarised as the following cyclic procedure:

1. An optimal generator schedule is decided for ensuring the whole day demand according to a forecasted hourly demand and system security and operational constraints.

2. During the 24h for which the schedule is drawn up, more accurate load forecasts are made at periodic intervals.

3. At fixed intervals of length δT the committed generating units are dispatched to meet the system load at a minimum cost. This dispatch provides the desired loading levels for committed units for interval δT.

4. During intervals δT the Automatic Generation Control (AGC) system adjusts the generation to match the load, until the next economic dispatch computation is performed, otherwise the system operates under free governor action.

Since the economic dispatch relates to an anticipated demand up to a very short time period ahead the forecasted demand will normally be very accurate, regardless of forecasting methodology. In addition, the AGC system will often have the capability to adjust the generation between dispatches in a manner that preserves economy inspite of deviations from forecasted demand. Thus, provided the demand forecast is of sufficient quality to permit the loading algorithm to determine unit start-up times correctly the economic dispatch function should ensure that the actual loading achieved is as close to the optimum as is practicable.

Therefore, the main aim of a forecast evaluation is to determine the effect of demand forecasts on unit commitment decisions. This will concentrate on forecasts with maximum lead time of 24h and the minimum lead time for periodic intervals within the 24h period. In Section 1.3.2 it was stated that the maximum unit lead time was 2h, so the minimum forecast lead time applicable here is 2h. This prediction scenario would require a prediction scheme in which the 24 hourly time divisions were grouped into 24×2h periods overlapping by 1h as shown in Fig 3.4.1.

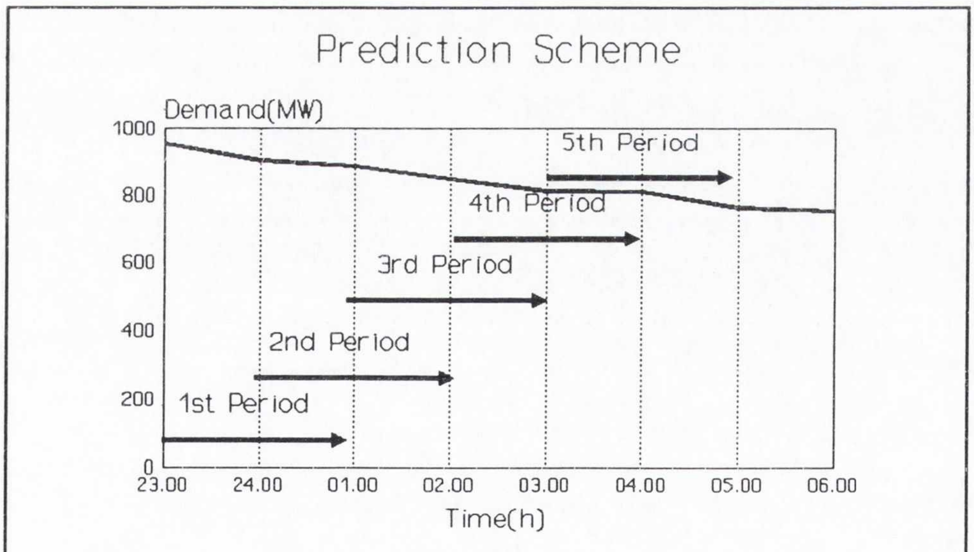


Figure 3.4.1 Prediction Scheme (2h ahead)

3.4.2 Methodology

Since thermal units require a lead time corresponding to run-up and synchronisation, load forecasting should correctly antipicate unit start-ups. When unit start-ups are not correctly anticipated they then have to be delayed, and other fast starting and more expensive units employed instead. The ability of univariate forecasts to foresee unit start-ups will now be evaluated.

Using forecasts for the 24h period a loading calculation is carried out to determine the 'predicted' unit commitments in the same period. If it is assumed from points (3) and (4) above that demand will be correctly dispatched, then the significance of forecasting can be illustrated by applying the forecasted commitment pattern to the actual demand profile for the load calculation. Forecasting deficit may then be evaluated by comparing this solution with the solution determined by running the loading calculation for the known demand profile (ideal forecasting).

A forecast deficit evaluation was performed for both worst and best (MAPE) predictions for winter, spring, summer and autumn. The loading calculations employed the same parameters as in Chapter One. To illustrate the methodology the worst case summer 24h ahead prediction (MAPE : 3.85%) is considered.

a. 'Ideal' Prediction

A normal 24-point loading calculation was performed, Figure 3.4.2, assuming that demand variation over the 24h period under consideration was ideally forecasted.

Fuel Costs = £182,535

b. Forecasted Unit Commitment Schedule

Another 24-point loading calculation was executed using 24h ahead forecasted demand data, Figure 3.4.3. The unit commitments shown here are the forecasted unit commitment schedule.

c. Actual Demand Profile with Forecasted Unit Commitments

The forecasted unit commitment schedule was then used in a loading calculation using actual demand data, Figure 3.4.4. Unit availability was controlled where necessary.

Fuel Cost = £182,987

Forecasting Deficit = £182,987 - £182,535

= £452

This is a 0.25% increase from ideal prediction.

Similar analysis was applied to the best and worst 24h ahead predictions for winter, summer, spring and autumn and the results are summarised in Table 3.3. From these results it is apparent that the increase in cost is not necessarily dependent upon MAPE but on the error distribution throughout the 24h period. Indeed, high forecast inaccuracy at times critical to unit commitment change results in increased costs.

In the previous analysis, a 24h ahead forecasted unit commitment schedule was applied to the actual demand data for the forecast period and the resulting forecast deficit calculated. However periodic forecasts within the 24h period can also be made and provided unit lead time is considered an improved schedule should be possible.

Run time 0 h 3 min 49 s

Reserve cover = 0.68

Fuel cost (£) = 182535.3

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
611.	1.0	134	0	0	134	0	0	0	134	134	73	0	0	0	0	0	0
572.	2.0	125	0	0	125	0	0	0	125	125	68	0	0	0	0	0	0
554.	3.0	121	0	0	121	0	0	0	121	121	66	0	0	0	0	0	0
535.	4.0	117	0	0	117	0	0	0	117	117	64	0	0	0	0	0	0
511.	5.0	112	0	0	112	0	0	0	112	112	61	0	0	0	0	0	0
500.	6.0	110	0	0	110	0	0	0	110	110	60	0	0	0	0	0	0
552.	7.0	121	0	0	121	0	0	0	121	121	66	0	0	0	0	0	0
693.	8.0	152	0	0	152	0	0	0	152	107	127	0	0	0	0	0	0
842.	9.0	145	0	0	180	50	0	0	169	139	157	0	0	0	0	0	0
912.	10.0	140	0	0	180	95	0	0	169	168	157	0	0	0	0	0	0
926.	11.0	163	0	0	180	71	0	0	169	168	157	0	0	15	0	0	0
938.	12.0	162	0	0	180	83	0	0	169	168	157	0	0	15	0	0	0
938.	13.0	162	0	0	180	84	0	0	169	168	157	0	0	15	0	0	0
897.	14.0	140	0	0	180	80	0	0	169	168	157	0	0	0	0	0	0
865.	15.0	141	0	0	180	50	0	0	169	165	157	0	0	0	0	0	0
873.	16.0	141	0	0	180	55	0	0	169	168	157	0	0	0	0	0	0
932.	17.0	162	0	0	180	78	0	0	169	168	157	0	0	15	0	0	0
961.	18.0	162	0	0	180	102	0	0	169	168	157	0	0	19	0	0	0
870.	19.0	141	0	0	180	51	0	0	169	168	157	0	0	0	0	0	0
770.	20.0	133	0	0	169	0	0	0	169	147	150	0	0	0	0	0	0
726.	21.0	148	0	0	159	0	0	0	159	99	157	0	0	0	0	0	0
715.	22.0	152	0	0	157	0	0	0	157	90	157	0	0	0	0	0	0
722.	23.0	150	0	0	158	0	0	0	158	95	157	0	0	0	0	0	0
687.	24.0	151	0	0	151	0	0	0	151	129	104	0	0	0	0	0	0

Figure 3.4.2 'Ideal' Prediction

Run time 0 h 4 min 0 s

Reserve cover = 0.68

Fuel cost (£) = 189623.9

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
615.	1.0	135	0	0	135	0	0	0	135	135	73	0	0	0	0	0	0
584.	2.0	128	0	0	128	0	0	0	128	128	70	0	0	0	0	0	0
575.	3.0	126	0	0	126	0	0	0	126	126	68	0	0	0	0	0	0
561.	4.0	123	0	0	123	0	0	0	123	123	67	0	0	0	0	0	0
534.	5.0	117	0	0	117	0	0	0	117	117	64	0	0	0	0	0	0
520.	6.0	114	0	0	114	0	0	0	114	114	62	0	0	0	0	0	0
575.	7.0	126	0	0	126	0	0	0	126	126	68	0	0	0	0	0	0
718.	8.0	151	0	0	157	0	0	0	157	92	157	0	0	0	0	0	0
875.	9.0	141	0	0	180	57	0	0	169	168	157	0	0	0	0	0	0
952.	10.0	162	0	0	180	50	**63**	0	169	168	157	0	0	0	0	0	0
966.	11.0	162	0	0	180	50	**77**	0	169	168	157	0	0	0	0	0	0
976.	12.0	162	0	0	180	50	**87**	0	169	168	157	0	0	0	0	0	0
980.	13.0	162	0	0	180	50	**91**	0	169	168	157	0	0	0	0	0	0
950.	14.0	162	0	0	180	50	**60**	0	169	168	157	0	0	0	0	0	0
923.	15.0	165	0	0	180	50	**50**	0	169	149	157	0	0	0	0	0	0
928.	16.0	164	0	0	180	50	**50**	0	169	155	157	0	0	0	0	0	0
976.	17.0	162	0	0	180	50	**87**	0	169	168	157	0	0	0	0	0	0
989.	18.0	161	0	0	180	50	**100**	0	169	168	157	0	0	0	0	0	0
884.	19.0	141	0	0	180	66	0	0	169	168	157	0	0	0	0	0	0
781.	20.0	132	0	0	168	0	0	0	169	161	149	0	0	0	0	0	0
744.	21.0	142	0	0	163	0	0	0	163	116	157	0	0	0	0	0	0
743.	22.0	142	0	0	163	0	0	0	163	114	157	0	0	0	0	0	0
753.	23.0	139	0	0	165	0	0	0	165	126	156	0	0	0	0	0	0
706.	24.0	153	0	0	155	0	0	0	155	86	155	0	0	0	0	0	0

Figure 3.4.3 Forecasted Unit Commitment Schedule

Reserve cover = 0.68

Fuel cost (£) = 182,986.7

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
611.	1.0	134	0	0	134	0	0	0	134	134	73	0	0	0	0	0	0
572.	2.0	125	0	0	125	0	0	0	125	125	68	0	0	0	0	0	0
554.	3.0	121	0	0	121	0	0	0	121	121	66	0	0	0	0	0	0
535.	4.0	117	0	0	117	0	0	0	117	117	64	0	0	0	0	0	0
511.	5.0	112	0	0	112	0	0	0	112	112	61	0	0	0	0	0	0
500.	6.0	110	0	0	110	0	0	0	110	110	60	0	0	0	0	0	0
552.	7.0	121	0	0	121	0	0	0	121	121	66	0	0	0	0	0	0
693.	8.0	152	0	0	152	0	0	0	152	107	127	0	0	0	0	0	0
842.	9.0	145	0	0	180	50	0	0	169	139	157	0	0	0	0	0	0
912.	10.0	166	0	0	180	50	50	0	169	137	157	0	0	0	0	0	0
926.	11.0	164	0	0	180	50	50	0	169	152	157	0	0	0	0	0	0
938.	12.0	163	0	0	180	50	50	0	169	166	157	0	0	0	0	0	0
938.	13.0	163	0	0	180	50	50	0	169	166	157	0	0	0	0	0	0
897.	14.0	168	0	0	180	50	50	0	169	120	157	0	0	0	0	0	0
865.	15.0	173	0	0	180	50	50	0	169	83	157	0	0	0	0	0	0
873.	16.0	172	0	0	180	50	50	0	169	93	157	0	0	0	0	0	0
932.	17.0	164	0	0	180	50	50	0	169	160	157	0	0	0	0	0	0
961.	18.0	162	0	0	180	50	71	0	169	168	157	0	0	0	0	0	0
870.	19.0	141	0	0	180	51	0	0	169	168	157	0	0	0	0	0	0
770.	20.0	133	0	0	169	0	0	0	169	147	150	0	0	0	0	0	0
726.	21.0	148	0	0	159	0	0	0	159	99	157	0	0	0	0	0	0
715.	22.0	152	0	0	157	0	0	0	157	90	157	0	0	0	0	0	0
722.	23.0	150	0	0	158	0	0	0	158	95	157	0	0	0	0	0	0
687.	24.0	151	0	0	151	0	0	0	151	129	104	0	0	0	0	0	0

Figure 3.4.4 Forecasted Unit Commitment Operation

	Winter Best	Winter Worst	Spring Best	Spring Worst	Summer Best	Summer Worst	Autumn Best	Autumn Worst
MAPE (%)	1.00	3.58	1.6	3.85	1.26	3.64	1.55	5.25
Deficit (£)	439	186	658	550	0	451	831	193
Deficit (%)	0.18	0.09	0.33	0.32	0.00	0.25	0.39	0.09

Table 3.3 Forecast Deficit (24h ahead)

For the summer case considered previously, a loading calculation was performed using the minimum possible forecast lead time (2h) prediction scheme mentioned earlier. As these forecasts are 2h ahead unit lead times are inherently accounted for. The forecasted unit commitments produced are shown in Figure 3.4.5 and if it is assummed that the load is properly dispatched then the forecasting deficit will be zero.

Run time 0 h 3 min 49 s

Reserve cover = 0.68

Fuel cost (£) = 184079.5

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
613.	1.0	134	0	0	134	0	0	0	134	134	73	0	0	0	0	0	0
583.	2.0	128	0	0	128	0	0	0	128	128	70	0	0	0	0	0	0
572.	3.0	125	0	0	125	0	0	0	125	125	68	0	0	0	0	0	0
551.	4.0	121	0	0	121	0	0	0	121	121	66	0	0	0	0	0	0
517.	5.0	113	0	0	113	0	0	0	113	113	62	0	0	0	0	0	0
499.	6.0	109	0	0	109	0	0	0	109	109	59	0	0	0	0	0	0
557.	7.0	122	0	0	122	0	0	0	122	122	66	0	0	0	0	0	0
702.	8.0	153	0	0	154	0	0	0	154	84	154	0	0	0	0	0	0
856.	9.0	143	0	0	180	50	0	0	169	154	157	0	0	0	0	0	0
932.	10.0	162	0	0	180	77	0	0	169	168	157	0	0	15	0	0	0
940.	11.0	162	0	0	180	86	0	0	169	168	157	0	0	15	0	0	0
944.	12.0	162	0	0	180	90	0	0	169	168	157	0	0	15	0	0	0
948.	13.0	162	0	0	180	94	0	0	169	168	157	0	0	15	0	0	0
919.	14.0	141	0	0	179	102	0	0	169	168	157	0	0	0	0	0	0
889.	15.0	141	0	0	180	71	0	0	169	168	157	0	0	0	0	0	0
886.	16.0	141	0	0	180	68	0	0	169	168	157	0	0	0	0	0	0
930.	17.0	162	0	0	180	75	0	0	169	168	157	0	0	15	0	0	0
945.	18.0	162	0	0	180	91	0	0	169	168	157	0	0	15	0	0	0
849.	19.0	144	0	0	180	50	0	0	169	146	157	0	0	0	0	0	0
759.	20.0	137	0	0	166	0	0	0	166	133	154	0	0	0	0	0	0
733.	21.0	146	0	0	161	0	0	0	161	106	157	0	0	0	0	0	0
723.	22.0	149	0	0	158	0	0	0	158	96	157	0	0	0	0	0	0
728.	23.0	148	0	0	160	0	0	0	160	101	157	0	0	0	0	0	0
684.	24.0	150	0	0	150	0	0	0	150	138	93	0	0	0	0	0	0

Figure 3.4.5 Forecasted Unit Commitment Schedule

Results corresponding to the 2h ahead predictions are provided in Table 3.4 and as would be expected this smaller forecast lead time reduces the forecasting deficit. The only anomaly occurs for the spring worst case prediction because although the MAPE falls with the 2h ahead prediction the forecasting deficit increases. The day in question is a Sunday and the adverse reaction of the 2h prediction to consumer volatility between 8.00 a.m. and 10.00 a.m. results in a less accurate commitment forecast.

	Winter Best	Winter Worst	Spring Best	Spring Worst	Summer Best	Summer Worst	Autumn Best	Autumn Worst
MAPE (%)	1.00	1.50	1.78	2.94	1.24	1.52	0.99	1.95
Deficit (£)	357	0	0	1087	0	0	643	0
Deficit (%)	0.14	0.00	0.00	0.62	0.00	0.00	0.31	0.00

Table 3.4 Forecast Deficit (2h ahead)

3.5 Conclusion

Univariate Box-Jenkins models were developed for each season of the year and forecasts produced. Although the model parameters varied the basic model form did not change significantly from season to season. Thus on-line operation could possibly be achieved by initially choosing a model off-line using a computer interactively, and then making on-line revisions to the model by tracking changes in the data and re-estimating the model parameters periodically.

Forecast accuracy was found to be dependent upon lead time used - forecast accuracy increased as lead time decreased. Consumer volatility towards weather changes seemed to be a major source of forecast error over larger lead times. This situation may be improved by the use of multivariate modelling incorporating weather variables.

The forecasts produced were then used to investigate the effect of forecast accuracy on system operational costs. Of course costs were influenced by the MAPE for each day but more important was the error distribution, i.e the time at which the error occurred. However, the univariate forecasting technique was found to provide sufficient accuracy to approach optimum operation over 2h ahead when used with an economic loading algorithm, assumming that an AGC system would make the necessary on-line adjustments to synchronised generation to account for forecasting errors.

REFERENCES

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4.1 Multivariate Analysis

In univariate modelling only past values of the dependent variable were used to model and forecast future values of that variable. Suppose that independent variables exist that are related to the dependent variable to be forecasted. The multivariate Box-Jenkins method, which is a natural extension of the univariate approach, attempts to relate these independent variables by means of transfer functions. These transfer function models are also known as vector autoregressive moving average (VARMA) models, or VARIMA models which include an integrated term for the modelling of non-stationary time series.

The multivariate model describes a single time series as a function of its own past values and the values of one or more independent input series. These models are inherently more useful than univariate ARIMA models when the single series is indeed a function of some related time series. The aim of multivariate modelling is to obtain a more accurate model than a univariate model of the observed system. Since the multivariate approach combines some of the characteristics of the univariate models and some of the characteristics of multiple regression analysis, it includes both time series and causal approaches. Box and

Jenkins' original text (1) discussed the single input - single output problem and since then the original theory has been expanded to discuss a wider range of transfer function models (2-8).

Figure 4.1 shows an open loop stochastic system. The output y(t) is presumed to be influenced by one or more input series $x_i(t)$. The input series, $x_i(t)$, exerts an influence on the output series via a transfer function, which distributes the impact of $x_i(t)$ over several future time periods. The objective of transfer function modelling is to determine a parsimonious model relating y(t) to $x_i(t)$ and n(t).

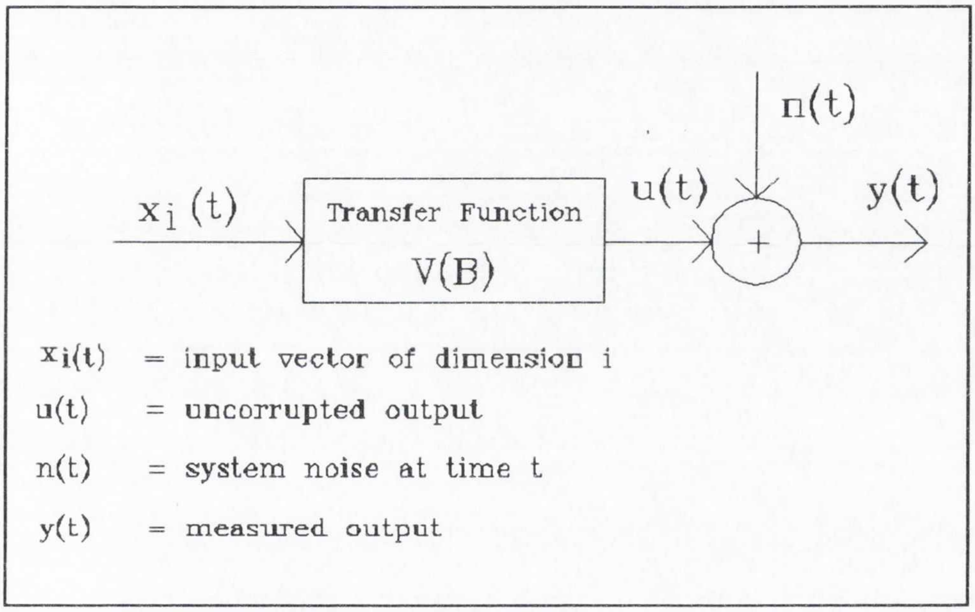

Figure 4.1 Open-loop Stochastic System

In control theory the transfer function of a linear system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input when all initial conditions are set to zero. If the operator $\upsilon(B)$ is the transfer function of the model of the above system, where the weights υ_i are called the impulse response function of the system, then:

$$y(t)=\sum_{i=1}^{m}\left[\upsilon_{0i}+\upsilon_{1i}x_i(t-1)+....+\upsilon_{ki}x_i(t-k)\right]+n(t) \tag{4.1.1}$$

$$y(t)=\sum_{i=1}^{m}\left(\upsilon_{0i}+\upsilon_{1i}B+\upsilon_{2i}B^2+....+\upsilon_{ki}B^k\right)x_i(t)+n(t)$$

$$y(t)=\sum_{i=1}^{m}\upsilon_{ki}(B)x_i(t)+n(t) \tag{4.1.2}$$

y(t) = the output series

$x_i(t)$ = the input series i

υ_{ki} = impulse response weights

n(t) = the disturbance input

m = the number of inputs

Equation 4.1.2 is the general form of the multi-input single-output (MISO) system. However, for ease of explanation it will be assumed that the system has one input and one output. Equation 4.1.2 can therefore be written as :

$$y(t)=\upsilon(B)x(t)+n(t) \tag{4.1.3}$$

This transfer function model can be more fully written as:

$$y(t)=\frac{\omega_i(B)}{\delta_i(B)}x_i(t-b_i)+n(t)$$
$$y(t)=\frac{\omega_i(B)}{\delta_i(B)}x_i(t-b_i)+\frac{\theta(B)}{\phi(B)}e(t) \tag{4.1.4}$$

where

$y(t)$	= the dependent output series
$\omega_i(B)$	= $\omega_{0i} - \omega_{1i}B - \omega_{2i}B^2 - .... -\omega_{si}B^s$, the numerator factor(s) on input series i
$\delta_i(B)$	= $1 - \delta_{1i}B - \delta_{2i}B^2 - .... - \delta_{ri}B^r$, the denominator factor(s) on the input series i
$x_i(t)$	= independent input series i
b_i	= pure time delay on input series i
$\theta(B)$	= $1 - \theta_1 B - \theta_2 B_2 - .... - \theta_q B^q$, the noise model MA factors
$\phi(B)$	= $1 - \phi_1 B - \phi_2 B^2 - .... - \phi_p B^p$, the noise model AR factors
$e(t)$	= noise series
B	= backward shift operator

The model building procedure for the multivariate model is based on a similar approach to that of the univariate model development with a few adjustments (3). The basic steps in developing a transfer function model are shown in Figure 4.2. There are four main stages and various sub-stages in the complete process of transfer function model building.

1. Identification
2. Estimation of the parameters
3. Diagnostic checks
4. Forecasting

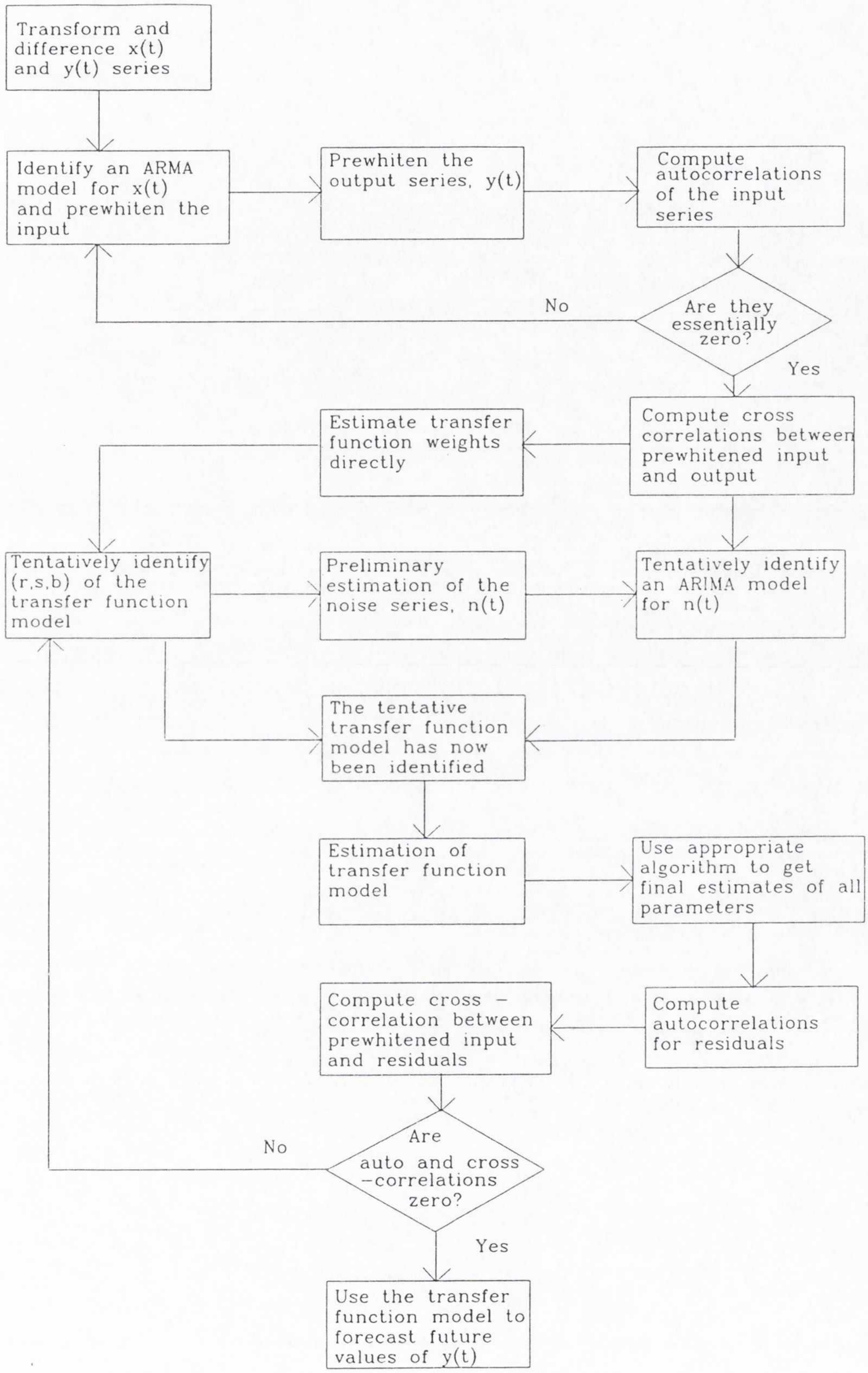

Figure 4.2 Multivariate Model Building Procedure

4.2 Identification

4.2.1 Data Preparation and Prewhitening

As with univariate identification, the initial status of the data to be modelled should be stationary and able to be described by a linear model. However if the raw data is not stationary, then appropriate differencing and transformation is required in order to remove non-stationarity and also the seasonality. It is assumed that the degree of differencing necessary to induce stationarity is achieved when the estimated autocorrelations and cross-correlations $r_{xx}(k)$, $r_{yy}(k)$ and $r_{xy}(k)$ of $x(t)=\nabla^d X(t)$ and $y(t)=\nabla^d Y(t)$ damp out quickly.

When using the transfer function of a system that transforms the input data into an output it is beneficial if the input data is as simple as possible. Therefore the input series is initially prewhitened - all known patterns are removed to leave just white noise. If the input series can be modelled as an ARIMA(p,0,q) process then,

$$\phi(B)\,x(t)=\theta(B)\,\alpha(t) \tag{4.2.1}$$

where

$\phi(B)$ = the AR operator

$\theta(B)$ = the MA operator

$\alpha(B)$ = white noise

(Note that there is no need for differencing in the ARIMA model because this should have already been taken care of previously.)

Then by rearranging terms in 4.2.1 the x(t) series can be converted into the $\alpha(t)$ series:

$$\alpha(t)=\frac{\phi(B)}{\theta(B)}x(t) \tag{4.2.2}$$

This is what is meant by prewhitening the x(t) series. If a prewhitening transformation is applied to the input series, then it is necessary to apply the same transformation to y(t) in order to preserve the integrity of the functional relationship since the transfer function that is being identified maps x(t) to y(t). Prewhitening is important on two accounts. Firstly it is necessary to induce stationarity for each stochastic series in the equation. Secondly, filtering the input and output series removes any intrarelationship so that the cross-correlation function reveals only the interrelationships. It should be noted that this transformation of y(t) does not necessarily convert y(t) to white noise. The 'prewhitened' output series will be known as β(t), where:

$$\beta(t)=\frac{\phi(B)}{\theta(B)}y(t) \tag{4.2.3}$$

The next stage in the identification procedure is to cross-correlate and autocorrelate the prewhitened input and output series. In univariate ARIMA modelling the autocorrelation coefficient was the key statistic in helping to identify the nature of the model. However in multivariate modelling the autocorrelation is not as important as the cross-correlation in identifying a tentative model. Extending the autocovariance function of equation 2.3.8 to the covariance between two variables x_t and y_t it can be defined as:

$$\gamma_{xy}(k)=\frac{1}{N}\sum_{t=1}^{N-k}(x_t-\bar{x})(y_{t+k}-\bar{y}) \tag{4.2.4}$$

The covariance $\gamma_{xy}(k)$ can be readily converted to the cross-correlation $\rho_{xy}(k)$ between x and y by dividing by $\sigma_x\sigma_y$. Therefore:

$$\rho_{xy}(k) = \frac{\gamma_{xy}(k)}{\left[\gamma_{xx}(0)\,\gamma_{yy}(0)\right]^{0.5}} = \frac{\gamma_{xy}(k)}{\sigma_x \sigma_y} \tag{4.2.5}$$

where the cross-correlation between x and y is defined as the association between the values of x at time t and the values of y at time t-k.

Again the sample cross-correlation, $r_{xy}(k)$ is only an estimate of the theoretical cross-correlation and may be obtained by substituting in equation 4.2.5 the estimates for $\gamma_{xy}(k)$, σ_x and σ_y:

$$\begin{aligned} r_{xy}(k) &= \hat{\rho}_{xy}(k) \\ c_{xy}(k) &= \hat{\gamma}_{xy}(k) \\ s_x(k) &= \hat{\sigma}_x(k) \\ s_y(k) &= \hat{\sigma}_y(k) \end{aligned}$$

The sampling distribution of the cross-correlations is very difficult to determine, but if two independent white noise series are considered the cross-correlation will be zero and the standard error can be approximated by $(1/N)^{0.5}$. This was extended to the case of two uncorrelated series, one being white noise, and for the cross-correlation of lag k, the standard error was approximated by:

$$se \approx (N-k)^{-0.5} \tag{4.2.6}$$

The cross-correlations are most important in model identification, so that this standard error formula becomes useful for detecting those cross-correlations that are significant.

Regular autocorrelations can also be computed for the prewhitened input and output series. For the prewhitened x_t series (namely the α_t series) there should not be any significant autocorrelations, but the prewhitened y_t series (namely the β_t series) may well include some pattern - precisely what should be expected from the transfer function.

As with the identification of the order of univariate models, it is important to identify tentatively the order of a multivariate model. A review of standard procedures for identifying the structure of a model was given in Section 2.3.2. These procedures, as well as those outlined, such as the sum of squares of the errors versus model order and the Akaike Information Criterion (AIC), also apply to multivariate models.

4.2.2 Estimation of Impulse Response Weights

Having prewhitened the input and output series and computed the cross-correlations between them the next step is to obtain a direct estimate for each of the impulse response weights. The prewhitened version of 4.1.3 can be written as:

$$\beta(t)=\upsilon(B)\alpha(t)+e(t) \tag{4.2.7}$$

where e(t) is the transformed noise series defined by:

$$e(t)=\frac{\phi(B)}{\theta(B)}n(t), \qquad \textit{see} \quad 4.1.4$$

Multiplying both sides of the equation by α(t-k) and taking expectations:

$$E[\alpha(t-k)\beta(t)]=\upsilon_0 E[\alpha(t-k)\alpha(t)]+\upsilon_1[\alpha(t-k)\alpha(t-1)]+....+E[\alpha(t-k)e(t)] \tag{4.2.8}$$

$$\gamma_{\alpha\beta}=\upsilon_k\gamma_{\alpha\alpha}(t-k)+0 \tag{4.2.9}$$

(since α and e are assumed to be independent)

Note that in equation 4.2.9 only the υ_k term appears because α(t-k) is independent of all other α(t) values. Rearranging 4.2.9 yields

$$\upsilon_k = \frac{\gamma_{\alpha\beta}(k)}{\sigma_\alpha^{\ 2}} = \frac{\rho_{\alpha\beta}(k)\sigma_\beta}{\sigma_\alpha} \tag{4.2.10}$$

However in practice the theoretical cross-correlation function $\rho_{\alpha\beta}(k)$ is not known and the estimates must be used instead. Therefore

$$\hat{\upsilon}_k = \frac{r_{\alpha\beta}(k)s_\beta}{s_\alpha} \tag{4.2.11}$$

where

$r_{\alpha\beta}$ = the estimated cross-correlation of the prewhitened input and output data

s_α and s_β = the estimated standard deviations of the prewhitened input and output data

k = lag time

The preliminary estimates of the impulse response weights $\hat{\upsilon}$ are generally insignificant but can provide a rough basis for selecting suitable operators $\delta(B)$ and $\omega(B)$ in the transfer function model.

4.2.3 Identification of Transfer Function Model Parameters

The three main parameters in the transfer function are r, s, and b, see equation 4.1.4, where r refers to the order of the $\delta(B)$ function, s refers to the order of the $\omega(B)$ function and b refers to the pure time delay.

The parameter 'b' is straight-forward to determine and can be obtained from the cross-correlation of the input and output. If the lag at which a statistically significant cross-correlation coefficient is observed is n, then the first estimate of the time delay is n-1. The parameter s indicates how long the output series continues to be influenced by new values of the input and r indicates the significant values of its own past, y(t-1), y(t-2), ..., y(t-r).

The bivariate transfer function model can be written in two general forms:

$$y(t)=\upsilon(B)x(t)+n(t)$$
$$y(t)=\frac{\omega(B)}{\delta(B)}x(t-b)+n(t)$$

By combining these two model forms the following identity is produced:

$$\upsilon(B)x(t)=\frac{\omega(B)}{\delta(B)}x(t-b) \tag{4.2.12}$$

or

$$(1-\delta_1 B-\delta_2 B^2-....-\delta_r B^r)(\upsilon_0+\upsilon_1 B+\upsilon_2 B^2+....)$$
$$=(\omega_0-\omega_1 B-....-\omega_s B^s)B^b$$

If the coefficients of B are matched, then by using estimates of the impulse responses, estimates of the coefficients δ_1, ..., δ_r and ω_1, ..., ω_s can be calculated:

$\hat{\upsilon}_j = 0$ for $j<b$

$\hat{\upsilon}_j = \delta_1\hat{\upsilon}_{j-1} + ... + \delta_r\hat{\upsilon}_{j-r} + \omega_0$ for $j=b$

$\hat{\upsilon}_j = \delta_1\hat{\upsilon}_{j-1} + ... + \delta_r\hat{\upsilon}_{j-r} + \omega_{j-b}$ for $j=b+1$, ..., $b+s$

$\hat{\upsilon}_j = \delta_1\hat{\upsilon}_{j-1} + ... + \delta_r\hat{\upsilon}_{j-r}$ for $j>b+s$

The above facts are often summarised in the form of three guiding principles that are aimed at helping a forecaster decide on appropriate values for r, s and b.

1. Until the b^{th} time lag the cross-correlations will not be significantly different from zero.

2. For s further time lags the cross-correlations will not show any clear pattern.

3. For r further time lags the cross-correlations will show a clear pattern.

For example, with r and s both equal to zero, the impulse response consists of a single value $\upsilon_b=\omega_0=g$, Figure 4.3. The output is proportional to the input displaced by b time intervals,

00b
$$y(t)=gx(t-b)$$
$$y(t)=\omega_0 B^b x(t)$$

Details of all first and second order transfer function models, i.e. all combinations of r=0, 1, 2 and s= 0, 1, 2, are given by Box and Jenkins (1).

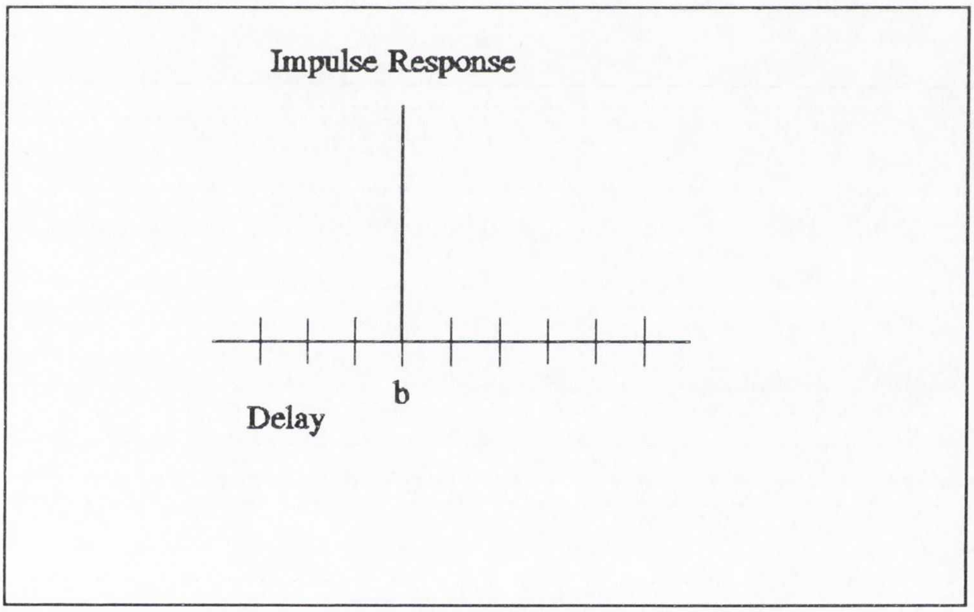

Figure 4.3 Impulse Response (0,0,b)

4.2.4 Examination of the Noise Series

The estimated impulse response weights enabled the calculation of preliminary estimates of the noise series. Consider again the general transfer function model:

$$\begin{aligned} y(t) &= \upsilon(B)x(t) + n(t) \\ n(t) &= \nabla^d N(t) \end{aligned} \tag{4.2.13}$$

Using preliminary estimates $\hat{\upsilon}(B)$ of the transfer function, an estimate of the noise series can be obtained by:

$$\begin{aligned} \hat{n}(t) &= y(t) - \hat{\upsilon}(B)x(t) \\ &= y(t) - \hat{\upsilon}_0 x(t) - \hat{\upsilon}_1 x(t-1) - \hat{\upsilon}_2 x(t-2) - .... \end{aligned} \tag{4.2.14}$$

Alternatively, $\hat{\upsilon}(B)$ may be replaced by the tentative transfer function model $\hat{\delta}^{-1}(B)\hat{\omega}(B)$. Thus

$$\hat{n}(t) = y(t) - \hat{\delta}^{-1}(B)\hat{\omega}(B)x(t-b) \tag{4.2.15}$$

After using equation 4.2.14 to estimate a noise series, the $\hat{n}(t)$ values are analysed in the conventional ARIMA manner to determine an appropriate ARIMA (p_n, 0, q_n) model to describe them. The autocorrelations and partial autocorrelations are found and the values p_n and q_n for the AR and MA processes, respectively, are chosen. In this manner, the $\phi(B)$ and $\theta(B)$ terms for this noise series, n(t) in equation 4.1.4, are obtained to give:

$$\phi(B)n(t) = \theta(B)e(t) \tag{4.2.16}$$

4.3 Estimation and Diagnostic Checking

4.3.1 Estimation of Model Parameters

For illustrative purposes, consider the bivariate case of equation 4.1.4,

$$y(t)=\frac{\omega(B)}{\delta(B)}x(t-b)+\frac{\theta(B)}{\phi(B)}e(t)$$

$$y(t)=\frac{(\omega_0-\omega_1B-....-\omega_sB^s)}{(1-\delta_1B-....-\delta_rB^r)}x(t-b)+\frac{(1-\theta_1B-....-\theta_qB^q)}{(1-\phi_1B-....-\phi_pB^p)}e(t) \qquad (4.3.1)$$

Previous analysis has outlined the identification of r, s, p, q and b. So having achieved the identification of these parameters, the next step is to estimate the parameters (ω_0, ..., ω_s), (δ_1, ..., δ_r), (θ_1, ..., θ_q) and (ϕ_1, ..., ϕ_p). Firstly, preliminary estimates are obtained using equation 4.2.12 which states explicitly the relationship between the impulse function, υ(B), and the left and right coefficient functions, δ(B) and ω(B). Since the impulse response weights can be estimated using equation 4.2.11, the parameters (ω_0, ..., ω_s) and (δ_1, ..., δ_r) can be estimated. Having obtained these preliminary estimates a non-linear least squares estimation procedure that is based on the Marquardt algorithm is used to proceed iteratively to better estimates.

Estimating the noise parameters (θ_1, ..., θ_q) and (ϕ_1, ..., ϕ_p) is achieved using the methods outlined in Chapter Two.

4.3.2 Diagnostic Checking

The next stage is to test the validity of the tentative models and identify those models which are inadequate. It should be noted that more than one model may be used to model the system accurately, but following Box and Jenkins' idea of parsimony, the model with the least number of parameters is chosen as the optimum model.

Initially the standard errors of the parameters are examined for significance using the t-ratio. The invertibility conditions of the model are then tested in order to check the invertibility of a transfer matrix. This is achieved by checking the invertibility of a lower order system. Several other tests are then used to determine model inadequacies, some of which are:

1. The autocorrelation function of the residuals $\hat{e}(t)$ from the fitted model.

2. The cross-correlation between the residuals $\hat{e}(t)$ and prewhitened input series $\alpha(t)$.

The aim in the sampling procedure is to obtain a model which generates white noise. Therefore the theoretical autocorrelation of the residuals and the cross-correlations of the residuals and the inputs should be zero. Consider the original form of the general model:

$$\begin{aligned} y(t) &= \frac{\omega(B)}{\delta(B)}x(t-b) + \frac{\theta(B)}{\phi(B)}e(t) \\ &= \upsilon(B)x(t) + \psi(B)e(t) \end{aligned} \tag{4.3.2}$$

Now suppose that an incorrect model is selected resulting in residuals $e_0(t)$, where

$$y(t) = \upsilon_0(B)x(t) + \psi_0(B)e_0(t)$$

Then,

$$e_0(t) = \psi_0^{-1}[\upsilon(B) - \upsilon_0(B)]x(t) + \psi_0^{-1}(B)\,\psi(B)e(t) \tag{4.3.3}$$

The residual errors $e_0(t)$ will be autocorrelated and they will also be cross-correlated with the prewhitened input series $\alpha(t)$, which indicates model inadequacy.

Now consider two cases:

1. When the transfer function model is correct, but the noise model is incorrect.

2. When the transfer function model is incorrect.

If $\upsilon_0(B)=\upsilon(B)$ but $\psi_0(B)\neq\psi(B)$, then the residual errors, $e_0(t)$'s, would not be correlated with the prewhitened input. The $e_0(t)$'s would however be autocorrelated and the form of the autocorrelation function could indicate the appropriate modification of the noise model. If the transfer function model was incorrect, i.e. $\upsilon_0(B)\neq\upsilon(B)$, then the $e_0(t)$'s would not only be cross-correlated with the x(t)'s, but they would also be autocorrelated. Even if the noise model was not correct, the cross-correlation analysis would indicate that the transfer function model requires modification. This aspect is clarified by considering the model after prewhitening. If the input and output are suitably transformed so that the input resembles white noise, the model may be written as:

$$\beta(t)=\upsilon(B)\alpha(t)+e(t) \tag{4.3.4}$$

where

$$\beta(t)=\phi(B)\theta^{-1}(B)y(t)$$
$$e(t)=\phi(B)\theta^{-1}(B)n(t)$$

Now consider

$$\begin{aligned} e_0(t)&=\beta(t)-\upsilon_0(B)\alpha(t) \\ &=[\upsilon(B)-\upsilon_0(B)]\alpha(t)+e(t) \end{aligned} \tag{4.3.5}$$

The cross-correlations between the $e_0(t)$'s the $\alpha(t)$'s measure the discrepancy between the correct and the incorrect impulse functions, as in:

$$\upsilon(k)-\upsilon_0(k)=\frac{\rho_{\alpha e_0}(k)\,\sigma_{e_0}}{\sigma_\alpha}, \quad k=0,1,2,.... \tag{4.3.6}$$

Another test for identifying the significance of a tentative model is determining whether a set of autocorrelations are significantly different from zero. This is based on the portmanteau lack-of-fit test which was outlined in Section 2.3.4. For the model to be adequate, the Q statistic, which was defined in equation 2.3.27, should be a normally distributed χ^2 distribution with m-p-q degrees of freedom.

$$Q=n\sum_{k=1}^{m} r_{ee}^{\,2}(k) \tag{4.3.7}$$

where

Q = Box-Pierce chi-squared statistic

n = the largest number of observations

m = the largest time lag considered

$r_{ee}(k)$ = the autocorrelations of the residuals at lag k.

The model here can also be checked by calculating the Box-Pierce statistic, known here as S, for the cross-correlation of the prewhitened input $\alpha(t)$ and the residual error e(t). If the model is to be adequate, then according to this test,

$$S=n\sum_{k=0}^{m} r_{\alpha e}^{\,2}(k) \tag{4.3.8}$$

should be approximately distributed as χ^2 with m-(r+s) degrees of freedom. Note that the number of degrees of freedom is independent of the number of parameters fitted to the noise model.

Those models which are chosen as being optimum are used for forecasting future values of the series of interest.

4.4 Intervention Modelling

4.4.1 Identifiable Isolated Events

Many time series may be influenced by identifiable, isolated events. Intervention analysis is a model development process for analysing this class of models. Intervention analysis is aimed at identifying the type of response a dependent variable will exhibit, given some step change in an independent variable (9). Box and Tiao (2) have shown the impulse response on the dependent variable for different types of changes. The objective is to identify an appropriate model similar to a transfer function model and to estimate its parameters. It is basically transfer function modelling with a stochastic output series and a deterministic input variable.

The values of the input variable are usually set to either zero or one , indicating 'off' or 'on'. For instance, a time series disturbed by a single event , such as a strike or a price change, could be modelled as a function of its own past values and a 'dummy' variable. The dummy input would be represented by a series of zeroes, with a value of one at the time period of intervention. A model of this form may better represent the time series under study. Intervention modelling can also be useful for 'what if' analysis - assessing the effect of possible deterministic changes to a stochastic time series.

The identification procedure for models of this type is as follows. The first step is to develop an ARIMA model for the output series. This model should be developed for the pre-intervention series. Forecasts from this model can then be plotted against the actual values in order to determine the nature of the effect of the intervention. The form of the transfer function between the output series and the intervention variable is suggested by this effect. The analyst then chooses the tentative transfer model form (2) and the tentative model noise form is obtained from the ARIMA model developed for the output series. Given the combined multiple input model form the next step is to proceed to the estimation/diagnostic checking phase.

4.4.2 Unknown Outliers

Intervention modelling can also be used to detect unknown outliers in a stochastic time series, Tsay (10). According to Tsay outliers 'are discrepant observations that look discordant from most observations in a data set' and so, it would be useful to employ models which reflected this fact. Outliers can be represented as intervention variables of the form: pulse, level shifts and seasonal pulses. The procedure for detecting the outlier variables is as follows:

1. Develop the appropriate ARIMA model for the time series under analysis.

2. Test the hypothesis that there is an outlier via a series of regressions at each time period.

3. Modify the residuals for any potential outliers and repeat the search until all possible outliers are discovered.

These outliers can then be included as intervention variables in a multiple input Box-Jenkins model. The noise model can be identified from the original series modified for the outliers. A possible application in this work could be the identification of the shift in demand data that occurs between BST and GMT. Also regular consumer response, for example television 'pick-up', may correspond to seasonal outlier pulses.

4.5 Example

This example describes an application of transfer function modelling to the relationship between two time series : an output series y(t) consisting of the hourly week day electrical load for the NIE system and an input series x(t), comprising the corresponding hourly temperatures (December 1989). (In the course of the modelling the output series is scaled by a factor of 1.0E+03)

Application of the usual ARIMA identification and fitting procedures yielded model forms for the input and output series:

Output Model Form

$$\nabla_{24}(1-0.89B)y(t)=(1-0.86B^{24})(1+0.22B^{120})e(t)$$

Input Model Form

$$(x(t)-4.14)(1-1.13B+0.18B^{2})=\alpha(t)$$

(Refer to equation 4.2.1)

Proceeding as in section 4.2.1, the prewhitened cross-correlation function is calculated by applying the temperature model to the seasonally differenced output series, that is

$$(1-1.13B+0.18B^{2})\nabla_{24}y(t)=\beta(t)$$

and then calculating the cross-correlation function between β(t) and α(t-k). For simplicity, in this example the cross-correlation function for the first few lags only is shown in Figure

4.4. The outstanding feature of the prewhitened cross-correlation function is a large negative value at lag zero, implying that an increase in temperature this hour decreases demand this hour and vice versa.

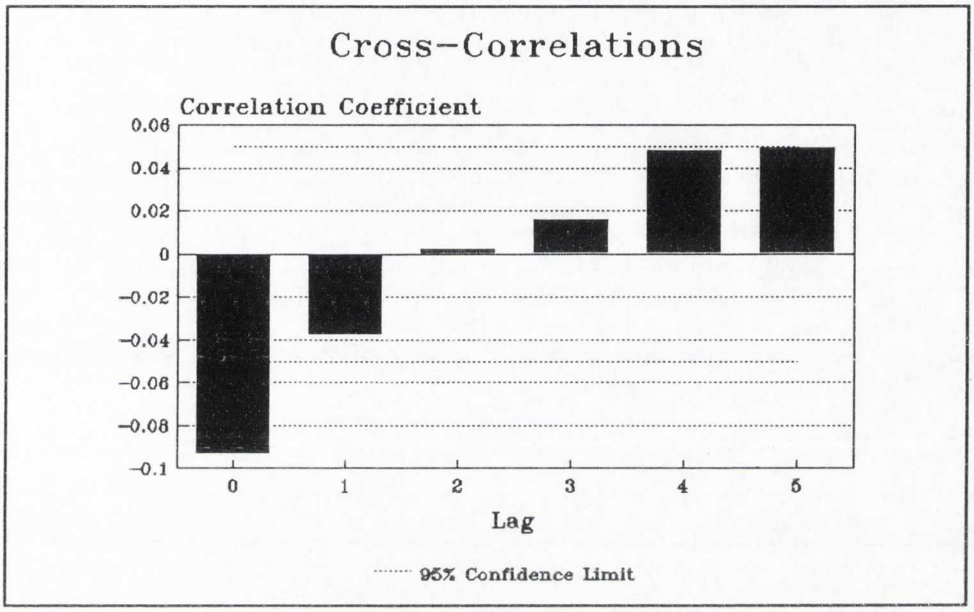

Figure 4.4 Cross-Correlations after Prewhitening

The next step is to compute the estimates of the impulse response weights, which are then examined for clues as to the appropriate transfer function model form. From equation 4.2.10 the impulse response weights or regression weights are proportional to the cross-correlations via the ratio of the standard deviations. For example at lag zero, the cross-correlation $r_{\alpha\beta}$ is -0.093 and the standard deviations of the input and output are 0.022676 and 0.53618 respectively. Therefore,

$$\hat{\upsilon}_0=(-0.093)\times\frac{0.022676}{0.53618}=-0.0039383$$

Table 4.1 lists the values of $\hat{\upsilon}_k$ versus k (the plot of these values is of the same form as Figure 4.4). Since there is a significant value at lag zero no time delay exists, i.e b=0. Also for r=2 lags a clear decay pattern occurs and hence there are no lags after the first significant lag that do not show any clear pattern, i.e. s=0. This evidence suggests r=2, s=0, b=0, a relationship of the form

$$(1-\delta_1 B-\delta_2 B^2)y(t)=\omega_0 x(t-0)+n(t)$$

Lag	Impulse Response Weights
0	-0.39383E-02
1	-0.15932E-02
2	0.79048E-04
3	0.67391E-03
4	0.20357E-02
5	0.20584E-02

Table 4.1 Impulse Response Function

<u>Suggested Tentative Model Form</u>

Number of output lag factors = 1

Number of input lag factors = 1

Delay (lag parameter) = 0

Output lag factor backorder powers = 1 and 2

Associated starting values = 0.88898 and -0.37944

Input lag factor backorder powers = 0

Associated starting values = -0.00394

(Starting values are obtained using equation 4.2.12)

Having obtained the tentative transfer function model the estimated noise series process is determined by applying the impulse weights to the original series. The noise series is then modelled in the conventional ARIMA manner. A first guess of the structure of the noise model is usually the univariate model for y(t), namely

$$\nabla_{24}(1-0.89B)n(t)=(1-0.86B^{24})(1+0.22B^{120})e(t)$$

The final estimated model is now given.

Noise Series

Differencing factors (order,degree) : 24,1

		Factor	Lag	Coefficient	T-ratio
1	AR	1	1	0.87839	88.95
2	MA	1	24	0.97757	182.28
3	MA	2	120	-0.2152	-4.21

Input Series

Differencing factors : None (Assumed mean of series is 4.088)
Value of lag parameter is 0

		Factor	Lag	Coefficient	T-ratio
4	Output Lag	1	1	1.0263	143.41
5	Output Lag	1	2	-0.40214	-3.58
6	Input Lag	1	0	-0.00268	-3.61

$$y(t)=\frac{-0.00268}{(1-1.03B+0.40B^2)}[x(t-0)-4.09]+\frac{(1-0.98B^{24})(1+0.22B^{120})}{\nabla_{24}(1-0.88B)}e(t)$$

Residual and Model Statistics

Sum of Squares of the Residuals : 0.088513

Mean Square of the Residuals : 0.22128E-03

R Squared Value : 0.99383

AIC : -8.4605

BIC : -8.4038

All of the parameters are invertible and significant.

Residual Analysis

Mean of the Residual Series : 0.0045998

Standard Deviation : 0.01434

Mean Divided by Standard Error of the Mean : 0.66516

As a first step in the diagnostic checking process the residual autocorrelations are plotted, Figure 4.5. The residual autocorrelation function looks for evidence of non-randomness in the residuals. Such non-randomness could be indicative of inadequacies in the transfer function and noise model. Next, a diagnostic check for transfer model sufficiency is performed by examining the cross-correlation function between the residuals from the transfer function model and the prewhitened input series, Figure 4.6. Forecasts were then produced using this multivariate model, enabling a comparison to be made with the corresponding univariate model. Figure 4.7 shows the percentage errors obtained using both models for a 24h ahead prediction. With multivariate prediction the MAPE for the day concerned falls from 1.44% to 1.00%.

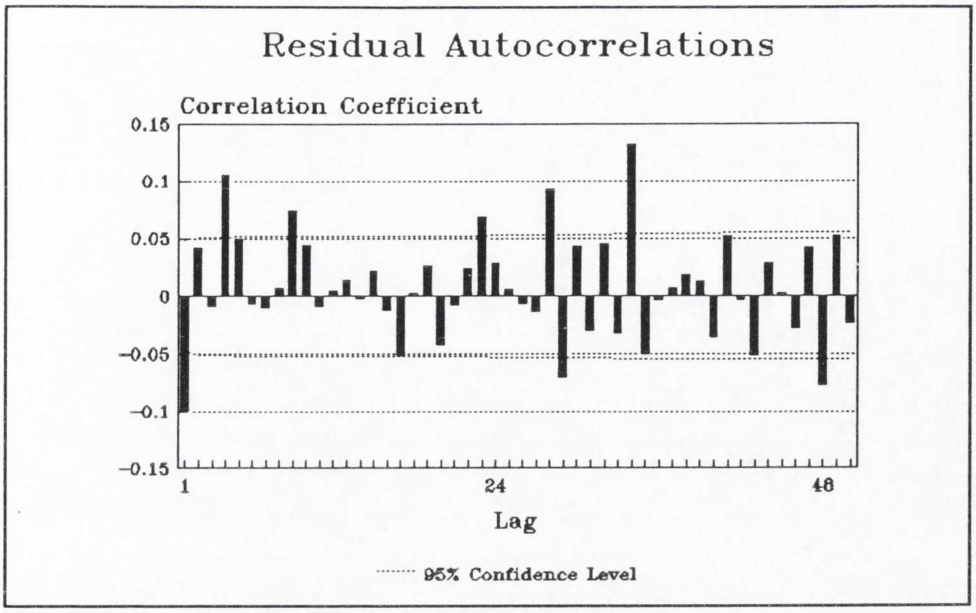

Figure 4.5 **Residual Autocorrelations**

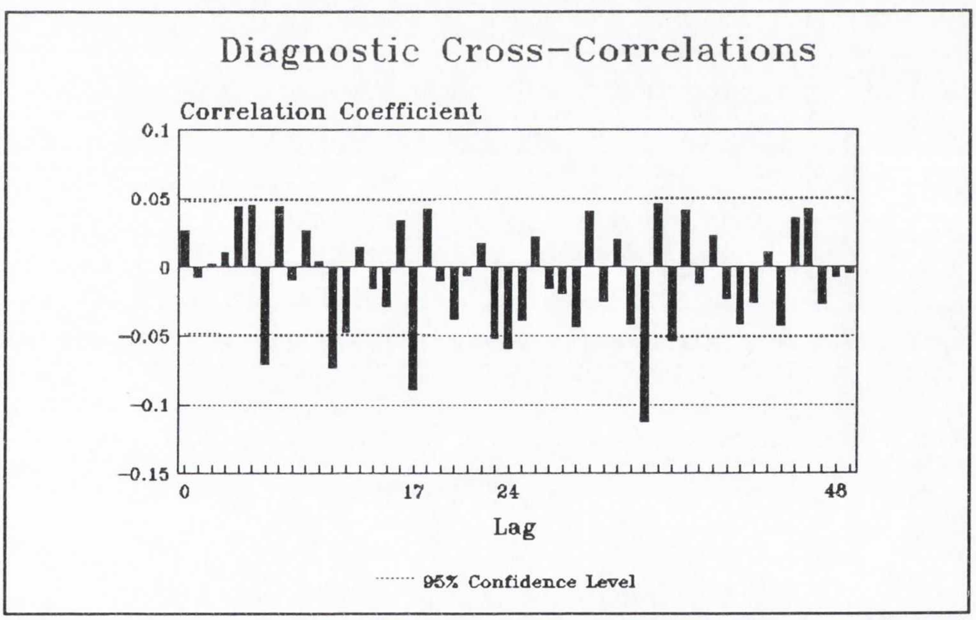

Figure 4.6 **Cross-Correlation between Residuals and Prewhitened Input**

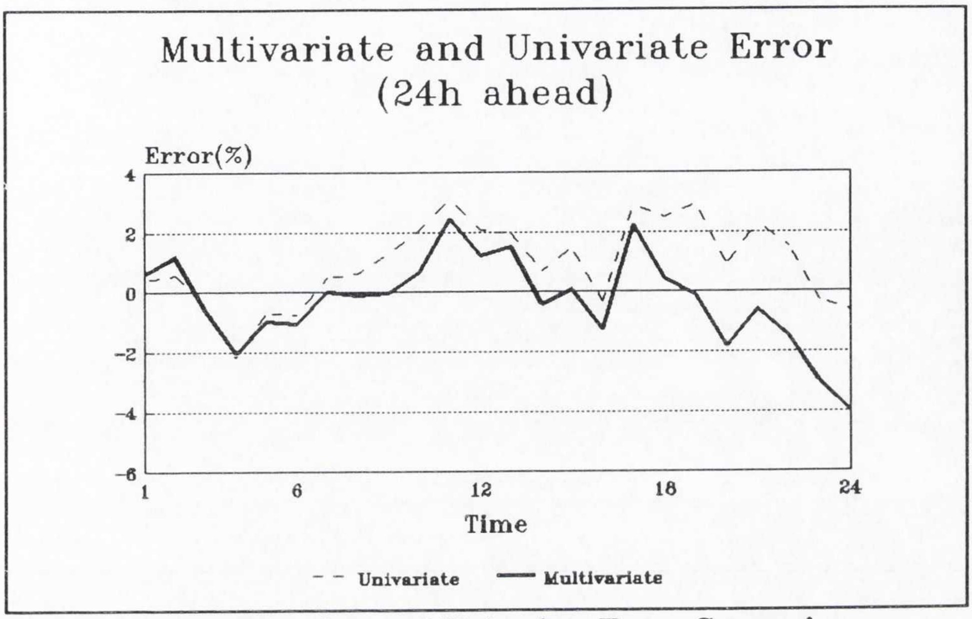

Figure 4.7 Multivariate and Univariate Error Comparison

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9. Tiao, G. C. and Box, G. E. P., 'Modelling multiple time series with applications', Journal of the American Statistical Association, Vol. 76, No.378, pp. 802-816, Dec. 1981.

10. Tsay, R. S., 'Time series model specification in the presence of outliers', Journal of the American Statistical Association, Vol. 76, No. 393, pp. 802-816, March 1986.

5.1 Climate of Northern Ireland

Two major factors are responsible for shaping the climate in the Province - proximity of the North Atlantic Ocean and the westerly atmospheric circulation of middle latitudes. Frontal systems and depressions with their associated meteorological patterns are a feature of this westerly circulation over the ocean and give a very variable climate here. Proximity of the ocean gives the climate a marked maritime character. The centres of most depressions crossing the North Atlantic pass to the north-west of the Province.

The relatively narrow range between summer and winter values of a number of meteorological elements is largely due to the moderating influence of the Atlantic Ocean. This oceanic influence tends to mask the effects of the annual variation in net radiation income available to the atmosphere at the latitude of Northern Ireland. Nevertheless a number of seasonal changes in addition to the more obvious change in temperature through the year tend to recur. Indeed some spells of weather tend to recur at approximately the same time each year.

5.2 The Relationship Between Weather and Demand

According to Troen and Petersen (1), 'weather is the totality of atmospheric conditions at any particular place and time - the instantaneous state of the atmosphere and especially those elements of the weather which directly affect living things. The elements of the weather are such things as temperature, atmospheric pressure, wind, humidity, cloudiness, rain, sunshine and visibility'. In this context fluctuations in weather or meteorological conditions have a vital role to play in accurately forecasting short-term load demand.

According to Heinnemann and Nordman (2), 'since the earliest days of the electric utility industry, system load has been responsive to changes in weather conditions'. Jager (3) maintains that the demand for electricity is particularly sensitive to weather. Superimposed on the marked regular pattern of diurnal, weekly and seasonal variations of electricity demand there are significant fluctuations attributable to meteorological factors. Indeed much effort has been expended on technologies that counter cold, excessive heat, ice, snow, winds and other manifestations of climate. Large components of weather sensitive load, such as those due to space heating, water heating, refrigeration and agriculture, can cause significant variations in the system load pattern. In many systems temperature is the most important weather variable. According to Buchanan (4), on a typical winter weekday in the U.K. a general fall in temperature of 1°C increases electricity demand by about 1.9 percent. Bolzern and Fronza (5) and Davies (6) have also found an improved regression between demand and temperature by taking past values into account, suggesting the existence of a certain delay in the reaction of some people to temperature changes. Furthermore a change in daylight illumination from a clear to a completely obscured sky can increase demand by as much as 5 percent at certain times of the year. Also, Barnett (7) states that a change in wind speed from 4 knots to 9 knots will increase demand by approximately 1 percent, assuming that there is no change in temperature. Maunder (8) indicates that weather is the dominant variable affecting domestic power consumption in New Zealand, and he estimates that in the absence of weather information, the cost of electricity generation (in 1988) would be increased by at least US$2 million a year. A weather manifestation peculiar to Northern Ireland is the fact that meteorological conditions near peripheral areas to windward are surmised to anticipate conditions at the major load centres. This downwind variation may have beneficial implications for demand prediction.

The total amount of generating plant is designed to meet the highest winter demand, but very large savings in fuel and money can be made by running just sufficient plant at any time to meet the consumer demand and by maintaining a calculated amount of spinning reserve against fluctuations caused by weather changes and other imponderables. So the inclusion of weather effects in any short-term forecasting scheme has a vital role to perform in the efficient operation of the system. Many authors have considered weather inputs, most notably temperature, in short-term prediction schemes. Ernoult (9), Papalexoulos (10), Poysti (11), Rakio (12), Vemuri (13) and Thompson (14) provide pertinent examples.

A prerequisite of the development of models that can effectively capture the historical relationship between demand and weather is the accurate monitoring and acquisition of relevant meteorological variables. The actual data acquisition system employed in this study and the weather variables monitored will now be discussed.

5.3 Data Acquisition

The meteorological data acquisition system comprises a personal computer (PC) and half-slot PC board with 80 pin custom VLSI chip. External sensors include an anemometer and wind vane, a temperature sensor and a light sensor. Atmospheric pressure is determined using an internally mounted sensor. Available software provides graphic representations of all weather events. Indeed a weather bulletin, which is updated instantaneously, is displayed on screen, showing all weather information simultaneously. Individual bulletins for each weather variable can also be selected for display. Data is stored on floppy disk every half-hour for atmospheric pressure, outside temperature, wind speed, wind gust, wind direction and light level. The half-hourly data for each day is stored in an individual data file. Figure 5.1 shows the data storage for a typical day.

11-02-91	BAROM MB	TEMP1 C	ILLUM LUX	CHILL C	SPEED KNOTS	GUST KNOTS	DIRECTION DEGREES	RAIN MM
00:00	987	7.3	0.0	-2.7	15	30	225	0
00:30	988	7.0	0.0	-1.6	12	23	225	0
01:00	988	7.0	0.0	-3.2	15	27	225	0
01:30	989	6.2	0.0	-5.3	17	31	225	0
02:00	990	5.6	0.0	-5.4	16	27	225	0
02:30	990	5.4	0.0	-5.1	15	25	225	0
03:00	991	5.3	0.0	-5.4	15	26	225	0
03:30	991	5.9	0.0	-3.9	14	26	225	0
04:00	991	5.8	0.0	-3.8	13	23	202	0
04:30	992	5.8	0.0	-4.7	15	25	225	0
05:00	992	5.8	0.0	-3.7	13	23	202	0
05:30	992	5.8	0.0	-4.4	14	27	202	0
06:00	992	5.8	0.0	-3.1	12	23	180	0
06:30	992	6.1	0.0	-3.7	14	25	180	0
07:00	992	6.1	0.0	-3.1	13	24	180	0
07:30	992	6.5	705.2	-1.6	11	23	180	0
08:00	992	6.8	1063.3	-1.7	12	23	180	0
08:30	992	6.9	2405.6	-0.3	10	22	157	0
09:00	992	6.9	2746.8	0.4	10	24	157	0
09:30	992	7.4	3396.6	-0.1	11	23	157	0
10:00	992	7.7	3978.0	0.6	10	22	157	0
10:30	992	7.7	2326.6	2.4	8	21	157	0
11:00	992	7.8	2795.1	2.9	8	19	157	0
11:30	991	7.6	1287.7	2.0	9	18	180	0
12:00	991	7.5	2051.3	1.9	9	18	180	0
12:30	991	7.4	3219.1	3.6	6	16	180	0
13:00	990	8.1	2795.1	5.4	5	14	112	0
13:30	990	7.9	1724.9	4.5	6	15	135	0
14:00	990	7.0	1079.6	1.1	9	18	225	0
14:30	989	5.9	1363.2	1.7	6	16	225	0
15:00	989	5.8	699.1	0.9	7	15	202	0
15:30	988	5.5	922.5	-0.1	8	17	202	0
16:00	988	5.4	367.4	3.6	3	10	157	0
16:30	987	5.1	0.0	4.2	3	8	135	0
17:00	987	5.4	0.0	4.4	3	9	135	0
17:30	986	5.2	0.0	4.4	3	9	135	0
18:00	986	5.0	0.0	2.9	4	12	135	0
18:30	986	4.9	0.0	-0.8	8	15	180	0
19:00	985	4.6	0.0	0.7	6	11	180	0
19:30	984	4.5	0.0	1.8	4	9	180	0
20:00	983	4.1	0.0	1.8	4	10	180	0
20:30	983	3.9	0.0	1.2	4	10	157	0
21:00	982	3.9	0.0	1.3	4	10	157	0
21:30	982	3.8	0.0	2.9	3	9	157	0
22:00	981	3.8	0.0	2.1	3	10	157	0
22:30	981	3.8	0.0	1.9	3	10	157	0
23:00	980	3.6	0.0	1.0	4	10	135	0
23:30	980	3.6	0.0	-0.9	6	12	180	0
LOWEST	979	3.4	0.0	-9.8	0	1	112	0
TIME	23:59	23:59	23:59	03:11	22:18	18:00	17:50	23:59
HIGHEST	993	8.6	8693.7	8.5	31	31	270	0
TIME	05:12	13:00	12:52	13:03	01:53	01:53	15:00	23:59
AVERAGE	988	5.9	244.3	-0.1	9	18	179	0

Figure 5.1 Data File (2nd November 1991)

The data acquisition system has been located at 'Castlereagh House' - NIE Control Centre, in Belfast, adjacent to the major load centre to the north-east of the province. The various meteorological sensors have been located on a roof top, with extensive cabling linking the sensors via marshalling boxes to a PC in the operations control room, accessible to control room engineers - see Figure 5.2.

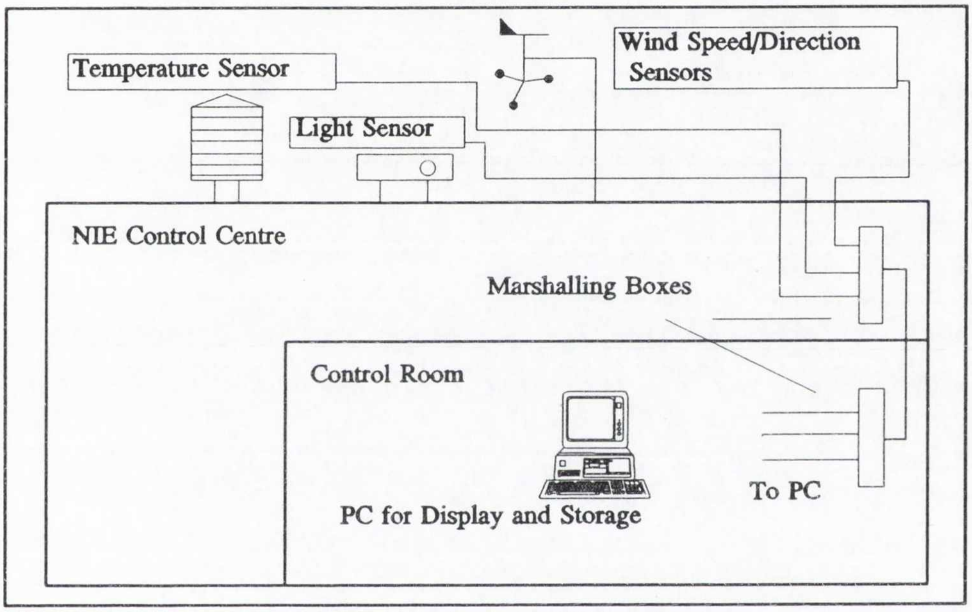

Figure 5.2 Schematic of Data Acquisition System

5.3.1 Temperature

Since temperature controls the heating demand it is necessary to measure the equilibrium temperature between a suitable sensor and free air. This is achieved using a bead thermistor - a semiconductor temperature sensor with a negative temperature coefficient. In Northern Ireland the air temperature seldom exceeds 25°C or drops more than 5°C below freezing and rarely remains below 0°C for more than 24 hours (15). So the range of this sensor, -70.5°C to 172.1°C, is more than adequate.

The sensor is housed in a louvred shelter to shield it from solar and terrestrial radiation and to allow sufficient ventilation for the temperature of air in contact with the thermometer to be at the same temperature as the air outside the shield. A purpose built shelter has been constructed at Q.U.B..

5.3.2 Wind

Wind allied to temperature is a significant factor in dissipating heat from buildings (wind-chill). Furthermore, high winds are normally associated with weather disturbances. So for meteorological purposes it is necessary to measure wind direction and magnitude to provide a complete specification of the wind. The direction is that from which the wind blows relative to true north and the magnitude is expressed as a wind speed. Wind speed is measured using a cup anemometer, in which speed is indicated by the speed of rotation of three cups rotating around a vertical axis. Direction is provided by a wind vane.

For Northern Ireland the mean (averaged over 20 years) wind speed over the year ranges from 7.5 m/s to less than 3.5 m/s (15). June to September are generally the months of lowest mean wind speed and strong winds are most frequent in the period November to March. While the foregoing remarks refer to average conditions, the wind regime shows considerable variations from year to year. Wind direction which manifests a south-westerly dominance (1) may have an application in an implicit forecasting scheme, where downwind demand variations in remote areas could provide an 'early warning' form of prediction.

The effect of ambient temperature and wind speed can be combined into a single parameter. This gives an indication about the discomfort level from a certain temperature and wind speed (16,17,18). This parameter is called the windchill index (WCI),

$$WCI = \frac{33-(10.45+10\sqrt{v}-v)(33-T)}{22.04}$$

where

WCI = wind chill equivalent temperature in °C

v = wind speed in m/s

T = temperature in °C

5.3.3 Light Level/Illumination

Davies (6) maintains that the lighting component of demand is determined by the daylight illumination received at the earth's surface. He asserts that cloud cover and atmospheric turbidity are the principal factors controlling the level of illumination. So, light levels during an average day depend to a large extent on weather conditions and season of the year. During the summer months there are more hours of daylight compared to winter, and skies tend to be clearer. Therefore in summer daylight levels are such that occupants can work by them alone on many occasions, while in winter greater use must be made of electric light because of fewer hours of daylight and mainly overcast skies.

It has also been shown that the probability of people switching lights on is most closely related to the minimum daylight illuminance on the working/living plane (19), Figure 5.3. Other work suggests the existence of a threshold level of this minimum daylight illuminance on the working plane of 300 lux (20) (lux is the unit of illumination, and is equal to 1 lumen per square metre).

A photo-conductive cell with a spectral response similar to that of the human eye was incorporated in the light sensor, Figure 5.4. The cell characteristic is shown in Figure 5.5.

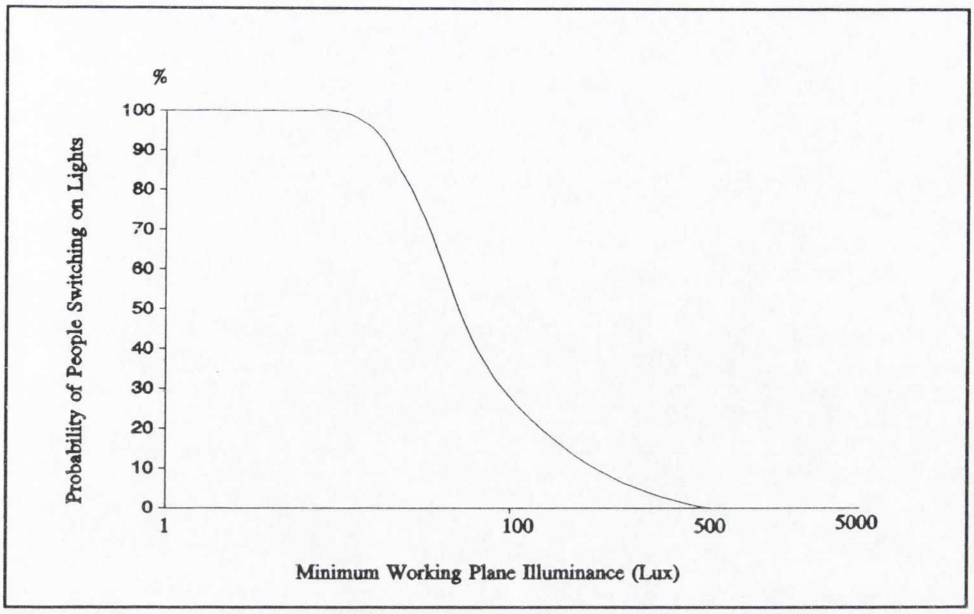

Figure 5.3 Switching Probability Relative to Ambient Light

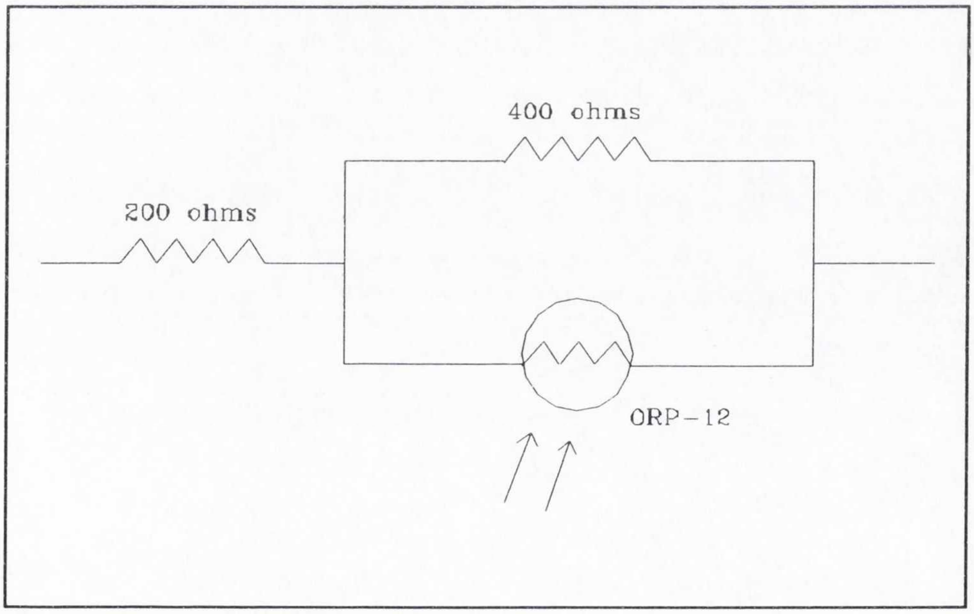

Figure 5.4 Light Sensor

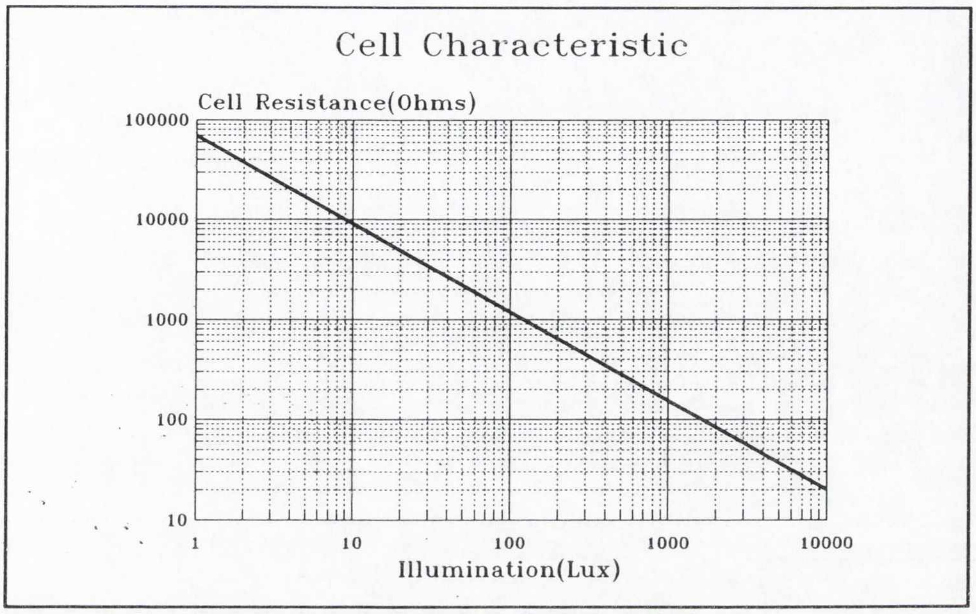

Figure 5.5 Photo-conductive Cell Characteristic

5.3.4 Atmospheric Pressure

Atmospheric pressure is simply the weight of the atmosphere bearing down on the earth's surface. However, due to the variability of the earth's topography, the force per unit area or pressure of the atmosphere is considered to describe this phenomena. The pressure reading is reduced to a standard level (mean sea level) by adding a reference value. The sea level barometric pressure reference for Belfast is 1027 mb.

Weather bulletins often refer to 'low' and 'high' weather fronts and systems. Each of these states have associated weather characteristics at different times of the year.

The data acquisition system outlined above has been installed and continuously monitoring data since the end of April 1991 to the present time. The data thus collated will now be used in Chapter Six to develop multivariate forecasting models.

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6.1 Introduction

A possible enhancement of univariate models is investigated using multivariate or transfer function modelling, which includes external variables understood to have an effect on electrical energy consumption. The external variables included are meteorological variables such as temperature and wind speed. Analysis is also carried out to determine if there is any cost benefit to be gained from employing multivariate Box-Jenkins time series forecasting.

6.2 Software Package

AutoBox Plus (ABP), which was used in the development of univariate models, was again employed in the development of multivariate models. A similar modelling approach was used for each, with the same algorithms used for identification, parameter estimation and statistical parameter calculation (1). The algorithm rigorously adheres to the process of model identification, estimation and diagnostic checking outlined in Chapter Four.

The tentative transfer function model is identified by examining the pattern in the estimated impulse weights, per the 'classical' method stated by Box and Jenkins. The tentative noise model and its preliminary parameter values are obtained by inputting the estimated noise series to the ARIMA development algorithm. Starting values for the coefficients in the transfer function model are computed using the routine devised by Box and Jenkins. These preliminary estimates are then input to a routine that computes the simultaneous parameter estimates via the Marquardt non-linear least squares estimation algorithm. The standard errors of the parameter estimates are then examined for necessity and the invertibility requirements of the model are tested. Model sufficiency checks include tests for both the transfer function model and the noise model. The residuals are examined for structure using normal guidelines. If the model passes all of the diagnostic checks, then it is deemed suitable for forecasting.

6.3 Weather Variables

Monitored meteorological data was available for temperature, wind speed, windchill, wind direction, illumination and pressure. Two other temperature related variables, heating degree-hour index and effective temperature, were considered as well.

The heating degree-hour (HDH) index provides a basis for establishing the relationship between fuel use (heating) and weather. The generally accepted definition of a degree-hour is the difference between a base temperature and the hourly outside temperature. The Department of Energy (2,3) indicates that an inside temperature of 18.3°C which is equivalent to an outside temperature of 15.5°C is considered comfortable for normal domestic purposes - a higher temperature is too warm, a lower temperature is too chilly. So,

$$HDH = 15.5°C - T_o$$

where T_o is the ambient temperature.

Effective temperature (4,5) takes account of the time lag of consumer response to temperature variations due to thermal storage in the fabric of buildings. In its simplest form effective temperature is defined by

$$T_e = (T_o + T_{e-24})/2$$

where T_o is the ambient temperature and T_e is the effective temperature.

Initially these variables were used to develop multivariate models for weekend demand data and subsequently for week-day demand data. The model forms are outlined in the next section.

6.4 Model Development

Applying the multivariate modelling procedures outlined in Chapter Four, models incorporating weather data as the input variables and NIE hourly total system load data as the output variable were developed. Model development was confined to two seasons, namely winter (Nov./Dec.) and autumn (Sept./Oct.). A data window of 6 weeks was examined for each season.

ARIMA models were first developed for the output and input series and then, using the iterative procedures of identification, estimation and diagnostic checking, multivariate transfer function models were produced. However, in the case of wind speed, wind direction, illumination and pressure the multivariate model was not significant and reduced to the univariate case. Only the temperature related variables proved to be significant for this form of multivariate modelling.

6.4.1 Weekend Models

Since Saturdays and Sundays have a demand profile distinct from other days of the week and because they possess an inherently larger domestic (weather sensitive) load, multivariate models were initially developed for weekends. Using winter data, models were generated with input variables of temperature, then effective temperature and finally heating degree-hours. Wind-chill was also found to be a significant input variable in autumn. The model forms are given below and the univariate and multivariate model parameter values are listed in Tables 6.1 and 6.2 respectively. (xi_m is the mean value of input series i)

Univariate Models

Output Series y(t) - Demand

Winter : $(1-B^{24})(1-\phi_1 B-\phi_4 B^4)(1-\phi_{24}B^{24})(1-\phi_{48}B^{48})y(t) = (1-\theta_{24}B^{24})e(t)$

Autumn : $(1-B^{24})(1-\phi_1 B)(1-\phi_{24}B^{24})y(t) = (1-\theta_2 B^2)(1-\theta_{24}B^{24})e(t)$

Input Series x1(t) - Temperature

Winter : $(1-\phi_1 B-\phi_2 B^2)(1-\phi_{24}B^{24})(x1(t)-x1_m) = e(t)$

Autumn : $(1-\phi_1 B-\phi_2 B^2)(x1(t)-x1_m) = (1-\theta_{24}B^{24})e(t)$

Input Series x2(t) - Effective Temperature

Winter : $(1-\phi_1 B-\phi_2 B^2)(1-\phi_{24}B^{24})(x2(t)-x2_m) = e(t)$

Autumn : $(1-\phi_1 B-\phi_2 B^2)(x2(t)-x2_m) = e(t)$

Input Series x3(t) - Heating Degree-Hours

Winter : $(1-\phi_1 B-\phi_2 B^2)(1-\phi_{24}B^{24})(x3(t)-x3_m) = e(t)$

Autumn : $(1-\phi_1 B-\phi_2 B^2-\phi_4 B^4)(x3(t)-x3_m) = e(t)$

Input Series x4(t) - Windchill Index

Autumn : $(1-\phi_1 B-\phi_4 B^4)(x4(t)-x4_m) = e(t)$

Multivariate Models - Winter

Input Series x1(t) - Temperature

$$y(t)=\omega_0(x1(t-b)-x1_m)+\frac{(1-\theta_{24}B^{24})}{(1-B^{24})(1-\phi_1 B-\phi_4 B^4)(1-\phi_{24}B^{24})(1-\phi_{48}B^{48})}e(t)$$

Input Series x2(t) - Effective Temperature

$$y(t)=\omega_0(x2(t-b)-x2_m)+\frac{(1-\theta_{24}B^{24})}{(1-B^{24})(1-\phi_1 B)(1-\phi_{24}B^{24})(1-\phi_{48}B^{48})}e(t)$$

Input Series x3(t) - Heating Degree-Hours

$$y(t)=\omega_0(x3(t-b)-x3_m)+\frac{(1-\theta_{24}B^{24})}{(1-B^{24})(1-\phi_1 B)(1-\phi_{24}B^{24})(1-\phi_{48}B^{48})}e(t)$$

Multivariate Models - Autumn

Input Series x1(t) - Temperature

$$y(t)=\omega_0(x1(t-b)-x1_m)+\frac{(1-\theta_{24}B^{24})}{(1-B^{24})(1-\phi_1 B)(1-\phi_{24}B^{24})}e(t)$$

Input Series x2(t) - Effective Temperature

$$y(t)=\frac{\omega_0}{1-\delta_1 B}(x2(t-b)-x2_m)+\frac{(1-\theta_{24}B^{24})}{(1-B^{24})(1-\phi_1 B)(1-\phi_{24}B^{24})}e(t)$$

Input Series x3(t) - Heating Degree-Hours

$$y(t) = \omega_0 (x3(t-b) - x3_m) + \frac{(1-\theta_{24}B^{24})}{(1-B^{24})(1-\phi_1 B)(1-\phi_{24}B^{24})} e(t)$$

Input Series x4(t) - Wind-Chill Index

$$y(t) = \frac{\omega_0}{1-\delta_1 B}(x4(t-b) - x4_m) + \frac{(1-\theta_2 B^2)(1-\theta_{24}B^{24})}{(1-B^{24})(1-\phi_1 B)(1-\phi_{24}B^{24})} e(t)$$

Parameter	y(t)		x1(t)		x2(t)		x3(t)		x4(t)	
	W	A	W	A	W	A	W	A	W	A
ϕ_1	0.83	0.76	1.17	1.32	1.14	1.5	1.17	1.33	-	0.99
ϕ_2	-	-	-0.20	-0.39	-0.18	-0.55	-0.20	-0.32	-	-
ϕ_4	0.09	-	-	-	-	-	-	-0.08	-	-0.08
ϕ_{24}	-0.96	-0.96	0.11	-	0.25	-	0.11	-	-	-
ϕ_{48}	-0.28	-	-	-	-	-	-	-	-	-
θ_2	-	-0.16	-	-	-	-	-	-	-	-
θ_{24}	-0.25	-0.61	-	-0.22	-	-	-	-	-	-
x_m	-	-	6.97	10.41	5.70	10.49	9.83	5.67	-	6.41

Table 6.1 Univariate Model Parameter Values (Weekend Series)

Parameter	x1(t)		x2(t)		x3(t)		x4(t)	
	W	A	W	A	W	A	W	A
ω_0	-2.62	-2.41	-4.30	-4.67	2.6	2.41	-	-8.91
δ_1	-	-	-	-0.78	-	-	-	0.59
b	2	22	2	9	2	22	-	14
ϕ_1	0.78	0.87	0.81	0.87	0.78	0.87	-	0.81
ϕ_4	0.10	-	-	-	0.10	-	-	-
ϕ_{24}	-0.96	-0.99	-0.96	-0.99	-0.97	-0.99	-	-0.98
ϕ_{48}	-0.27	-	-0.26	-	-0.27	-	-	-
θ_2	-	-	-	-	-	-	-	-0.17
θ_{24}	-0.34	-0.53	-0.36	-0.53	-0.34	-0.53	-	-0.50
x_m	6.58	10.50	5.70	10.81	10.22	5.00	-	6.41

Table 6.2 Multivariate Model Parameter Values (Weekend Series)

6.4.2 Week-Day Models

Since Monday to Friday have a similar demand profile, univariate and subsequently multivariate time series models were developed for the week-day period. For both the winter and summer seasons only temperature, effective temperature and the heating degree-hour index were found to be significant input variables. The resultant model forms are now outlined and the corresponding univariate and multivariate model parameter values are given in Tables 6.3 and 6.4.

Univariate Models

<u>Output Series y(t) - Demand</u>

Winter : $(1-B^{24})(1-\phi_1B-\phi_2B^2-\phi_3B^3)(1-\phi_{120}B^{120})y(t) = (1-\theta_{24}B^{24})e(t)$

Autumn : $(1-B^{24})(1-\phi_1B)(1-\phi_{120}B^{120})y(t) = (1-\theta_{24}B^{24})e(t)$

<u>Input Series x1(t) - Temperature</u>

Winter : $(1-\phi_1B-\phi_3B^3)(x1(t)-x1_m) = (1-\theta_{24}B^{24})e(t)$

Autumn : $(1-\phi_1B-\phi_3B^3)(x1(t)-x1_m) = e(t)$

Input Series x2(t) - Effective Temperature

Winter : $(1-\phi_1B-\phi_3B^3)(1-\phi_{24}B^{24})(x2(t)-x2_m) = (1-\theta_{24}B^{24})e(t)$

Autumn : $(1-\phi_1B-\phi_2B^2-\phi_3B^3)(x2(t)-x2_m) = e(t)$

Input Series x3(t) - Heating Degree-Hours

Winter : $(1-\phi_1B-\phi_3B^3)(1-\phi_{24}B^{24})(x3(t)-x3_m) = e(t)$

Autumn : $(1-\phi_1B-\phi_3B^3)(x3(t)-x3_m) = e(t)$

Multivariate Models - Winter

Input Series x1(t) - Temperature

$$y(t)=(\omega_0-\omega_1B-\omega_3B^3)(x1(t-b)-x1_m)+\frac{(1-\theta_{24}B^{24})}{(1-B^{24})(1-\phi_1B-\phi_2B^2-\phi_3B^3)(1-\phi_{120}B^{120})}e(t)$$

Input Series x2(t) - Effective Temperature

$$y(t)=\omega_0(x2(t-b)-x2_m)+\frac{(1-\theta_{24}B^{24})}{(1-B^{24})(1-\phi_1B-\phi_2B^2-\phi_3B^3)(1-\phi_{120}B^{120})}e(t)$$

Input Series x3(t) - Heating Degree-Hours

$$y(t)=\frac{\omega_0}{(1-\delta_1B)}(x3(t-b)-x3_m)+\frac{(1-\theta_{24}B^{24})}{(1-B^{24})(1-\phi_1B-\phi_2B^2-\phi_3B^3)(1-\phi_{120}B^{120})}e(t)$$

Multivariate Models - Autumn

Input Series x1(t) - Temperature

$$y(t)=\frac{(\omega_0-\omega_1 B-\omega_2 B^2)}{(1-\delta_1 B)}(x1(t-b)-x1_m)+\frac{(1-\theta_{24}B^{24})}{(1-\phi_1 B)(1-\phi_{120}B^{120})}e(t)$$

Input Series x2(t) - Effective Temperature

$$y(t)=(\omega_0-\omega_1 B)(x3(t-b)-x3_m)+\frac{(1-\theta_{24}B^{24})}{(1-\phi_1 B)(1-\phi_{120}B^{120})}e(t)$$

Input Series x3(t) - Heating Degree-Hours

$$y(t)=\frac{(\omega_0-\omega_1 B-\omega_2 B^2)}{(1-\delta_1 B)}(x3(t-b)-x3_m)+\frac{(1-\theta_{24}B^{24})}{(1-\phi_1 B)(1-\theta_{120}B^{120})}e(t)$$

Parameter	y(t)		x1(t)		x2(t)		x3(t)	
	W	A	W	A	W	A	W	A
ϕ_1	1.01	0.91	1.10	1.20	1.11	1.36	1.11	1.20
ϕ_2	-0.19	-	-	-	-	-0.19	-	-
ϕ_3	0.10	-	-0.16	-0.30	-0.17	-0.26	-0.16	-0.29
ϕ_{24}	-	-	-	-	0.10	-	0.10	-
ϕ_{120}	0.40	0.57	-	-	-	-	-	-
θ_{24}	0.73	0.51	-0.10	-	-0.53	-	-	-
x_m	-	-	4.60	11.30	4.53	11.20	10.84	4.19

Table 6.3 Univariate Model Parameter Values (Week-Day Series)

Parameter	x1(t)		x2(t)		x3(t)	
	W	A	W	A	W	A
ω_0	-2.95	-1.37	-3.40	-2.57	2.94	1.37
ω_1	1.85	2.87	-	2.42	-	2.87
ω_2	-	1.50	-	-	-	1.50
ω_3	1.97	-	-	-	-	-
δ_1	-	-0.72	-	-	0.56	-
b	0	0	1	0	0	0
ϕ_1	0.94	0.93	0.97	0.91	0.96	0.90
ϕ_2	-0.14	-	-0.17	-	-0.17	-
ϕ_3	0.12	-	0.09	-	0.12	-
ϕ_{120}	0.39	0.58	0.52	0.60	0.44	0.58
θ_{24}	0.77	0.50	0.74	0.51	0.74	0.50
x_m	4.69	11.27	4.66	11.12	10.81	4.23

Table 6.4 Multivariate Model Parameter Values (Week-Day Series)

6.5 Multivariate Forecasts

Results for two consecutive winter weeks provide a representative insight to the multivariate results generated. 24h ahead forecasts were determined for each day in the two week period. The multivariate model relating hourly temperature to demand was the most adequate and accurate model in the case of week-day data. For weekend data however, hourly effective temperature proved foremost. The first week under consideration manifests an increase in demand from the previous week, whereas the second week examined follows the demand level preponderant in the first week. This trend is reflected in the MAPE pertaining to each week i.e. in this first week accuracy is relatively poor and in the second week accuracy is improved. Univariate and multivariate results for the two week period are tabulated in Table 6.5 with the improvement in accuracy of the multivariate techniques shown in brackets. The package allows for two methods of obtaining forecast values of the input series, which are of course needed to obtain forecast values of the output series. Firstly, the forecast values of the input series can be generated via the ARIMA model developed for the input series. Alternatively the actual values for the input variables may be entered. The output series forecast accuracies produced by each method are also listed in Table 6.5.

Generally the multivariate models improve the MAPE for the 24h ahead prediction. Indeed, using temperature as an input variable improves forecast accuracy by 50% on the final Sunday of the two week period, see Figures 6.1 and 6.2. However, this is not always the case. For example, on the final Wednesday considered the multivariate technique proves to be less accurate (60%) than the univariate prediction. Figure 6.3 shows the 24h ahead univariate and multivariate predictions for this day. The corresponding temperature variation is shown in Figure 6.4 and it is evident that after 7.00 p.m. there is an increase in temperature. This increase is taken into account by the multivariate method resulting in a relative decrease in demand prediction. However there is not a corresponding consumer reaction to the temperature increase. This example highlights the irregularity of temperature induced consumer demand change.

Also improved accuracy is achieved using actual temperature inputs as opposed to forecasted temperature inputs, as would be expected.

Day	Univariate (MAPE)	Multivariate (MAPE)	
		Actual Temp	Forecast Temp
Monday	2.89	2.04 (29%)	2.30 (20%)
Tuesday	2.68	1.87 (30%)	2.33 (13%)
Wednesday	2.67	1.72 (36%)	2.66 (0%)
Thursday	0.99	1.77 (-78%)	0.97 (2%)
Friday	2.19	1.55 (29%)	1.55 (29%)
Saturday	1.24	1.11 (10%)	1.30 (-5%)
Sunday	1.25	1.13 (10%)	1.20 (4%)
Monday	1.67	1.24 (26%)	1.23 (26%)
Tuesday	1.06	1.04 (2%)	0.92 (13%)
Wednesday	0.73	1.17 (-60%)	0.83 (-14%)
Thursday	1.56	1.41 (10%)	1.52 (3%)
Friday	2.28	1.73 (24%)	1.52 (33%)
Saturday	1.91	1.74 (9%)	1.80 (6%)
Sunday	2.9	1.44 (50%)	1.90 (34%)

Table 6.5 Forecast Results

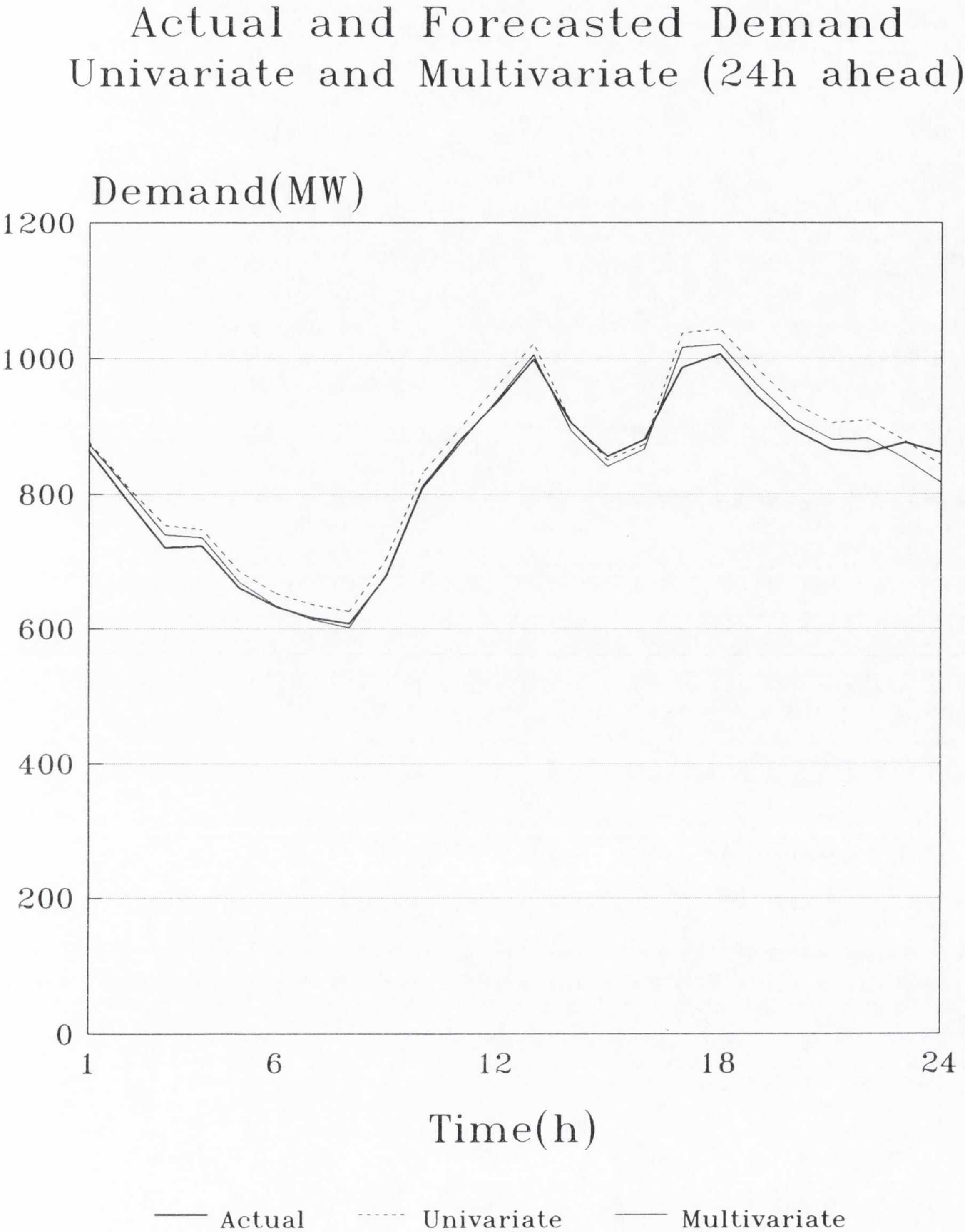
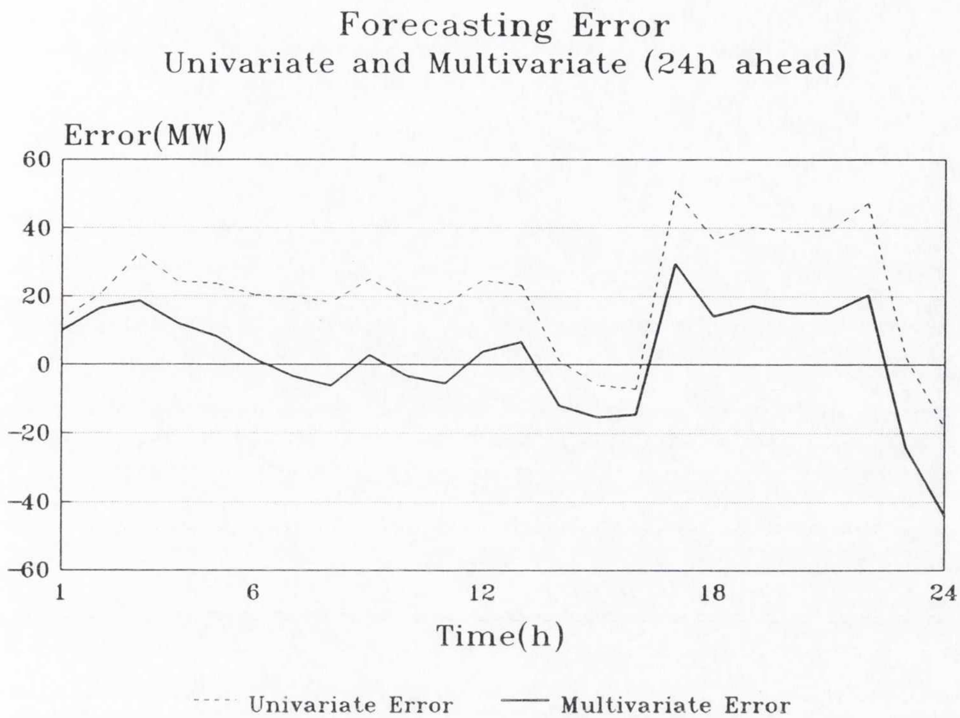


Figure 6.1 Actual and Forecasted Demand - Sunday (24h ahead)



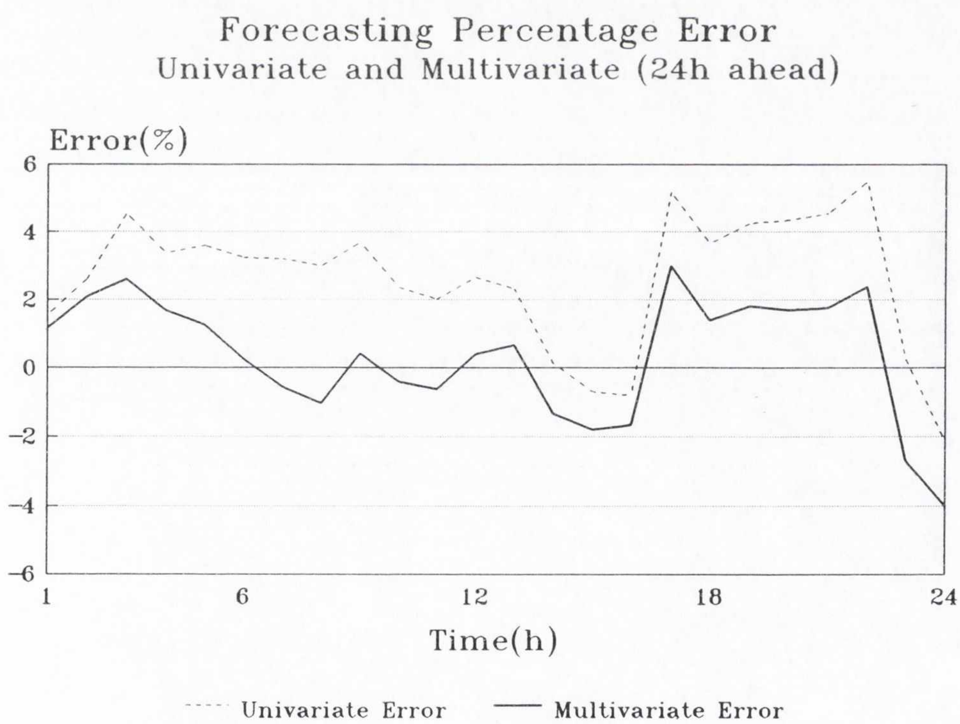


Figure 6.2 Actual and Percentage Errors - Sunday (24h ahead)

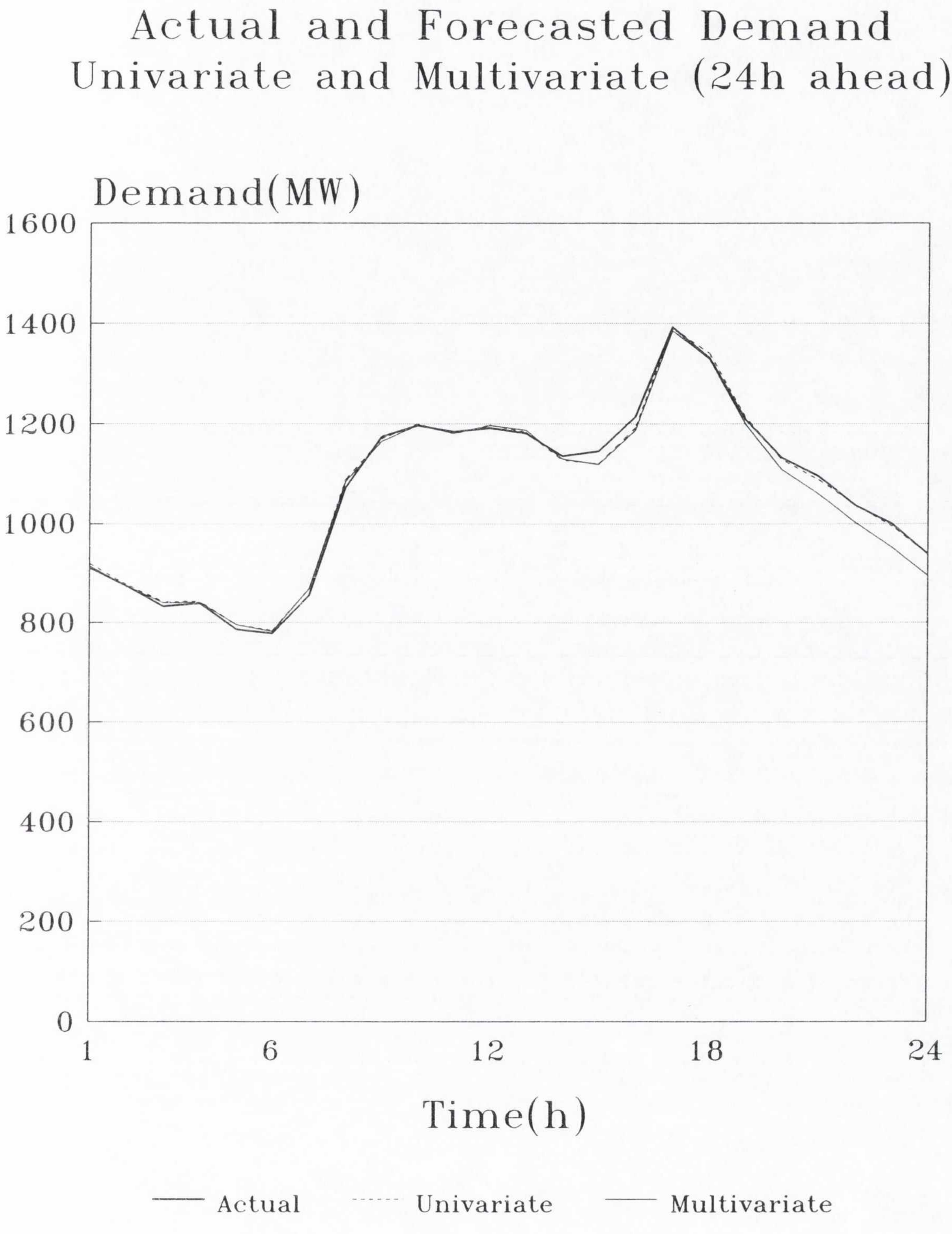
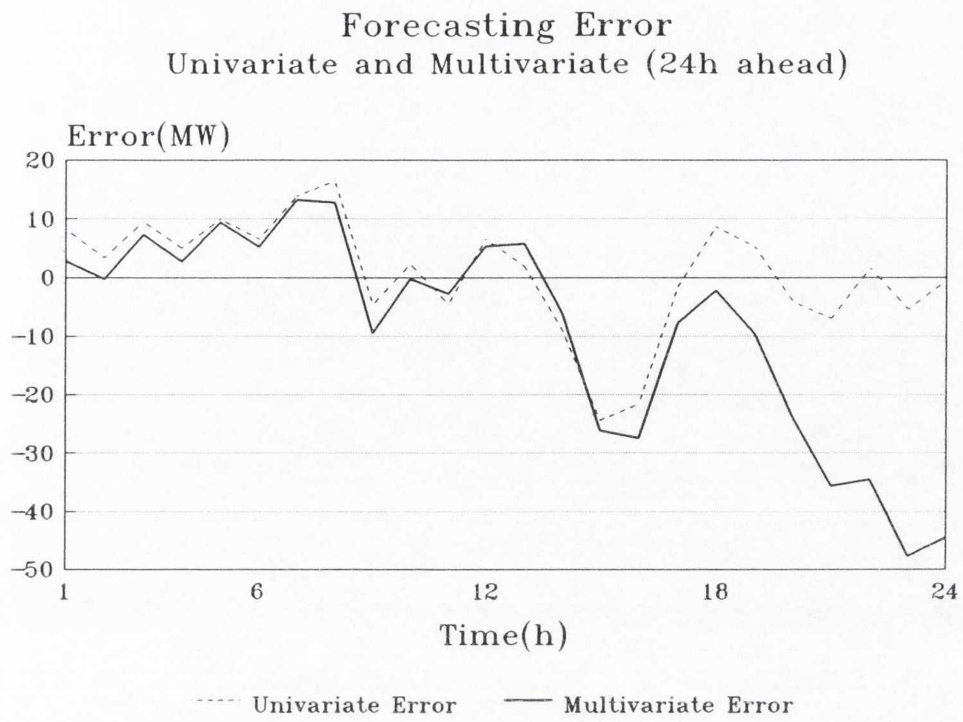


Figure 6.3 Actual and Forecasted Demand - Wednesday (24h ahead)



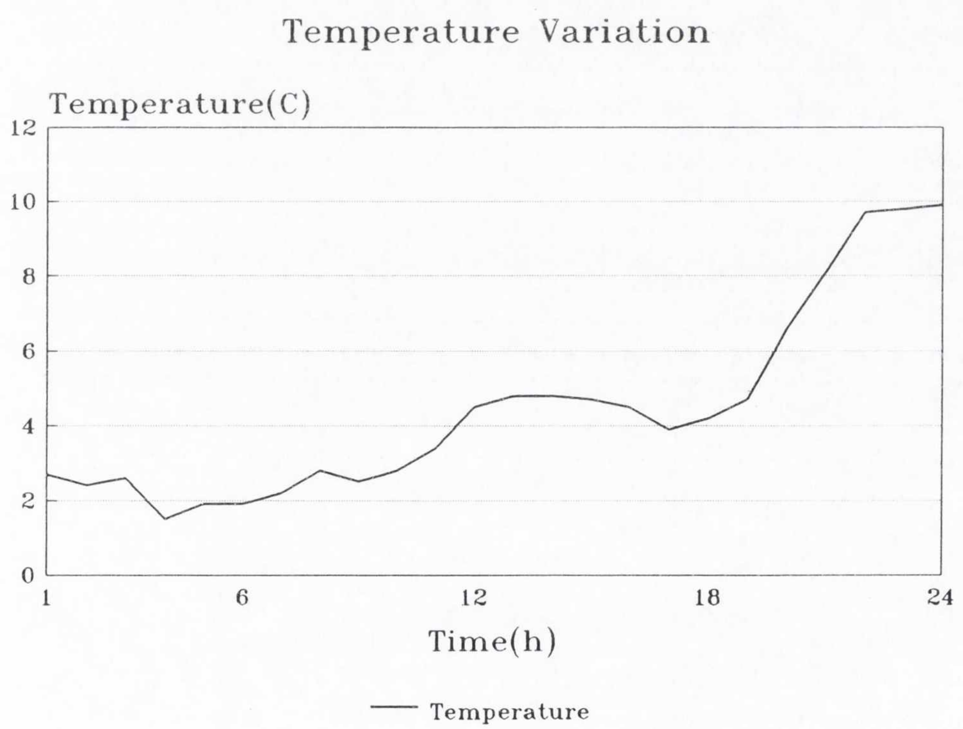


Figure 6.4 Error and Temperature Variation

6.6 Forecasting Deficit

As mentioned in Chapter Three, the main aim of a forecast evaluation is to determine the effect of demand forecasts on unit commitment decisions.

Since thermal units require a lead time corresponding to run-up and synchronisation, load forecasting should correctly anticipate unit start-ups. The possible enhancements in the ability of forecasts to foresee start-ups using multivariate forecasts will now be considered.

Using univariate and multivariate forecasts for the 24h period, a loading calculation is effected to determine the 'predicted' unit commitments in the same period and if it is assumed that the demand will be correctly dispatched, then the relative significance of both forecasting methods can be shown by applying the forecasted commitment pattern to the actual demand profile for the load calculation. Forecasting deficit may then be evaluated by comparing this solution with the solution determined by running the loading calculation for the known demand profile - ideal forecasting.

A forecast deficit evaluation was performed to ascertain the maximum possible improvement using multivariate forecasting. This relates to the 24h ahead prediction for the multivariate method which achieved a 50% improvement in accuracy. The loading calculations employed the same parameters as in Chapters One and Three.

a. 'Ideal' Prediction

A normal 24-point loading calculation was performed, Figure 6.5, assuming that demand variation over the 24h period under consideration was ideally forecasted.

Fuel Costs = £199,125

b. Univariate Prediction (MAPE : 2.90%)

i. Forecasted Unit Commitment Schedule

Another 24-point load calculation was executed with 24h ahead univariate forecasted demand data, Figure 6.6. The unit commitments shown here are the forecast unit commitment schedule.

ii. Actual Demand Profile with Forecasted Unit Commitments

The forecasted unit commitment schedule was then used in a loading calculation using actual demand data, Figure 6.7. Unit availability was controlled where necessary.

Fuel Cost = £199,385

Forecasting Deficit = £199,385 - £199,125

= £260

This is a 0.13% increase from ideal prediction.

Run time 0 h 4 min 13 s

Reserve cover = 0.68

Fuel cost (£) = 199125.3

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
870.	1.0	141	0	0	180	51	0	0	169	168	157	0	0	0	0	0	0
828.	2.0	147	0	0	180	50	0	0	169	122	157	0	0	0	0	0	0
756.	3.0	138	0	0	166	0	0	0	166	130	155	0	0	0	0	0	0
722.	4.0	150	0	0	158	0	0	0	158	96	157	0	0	0	0	0	0
693.	5.0	152	0	0	152	0	0	0	152	109	126	0	0	0	0	0	0
648.	6.0	142	0	0	142	0	0	0	142	142	77	0	0	0	0	0	0
625.	7.0	137	0	0	137	0	0	0	137	137	75	0	0	0	0	0	0
613.	8.0	134	0	0	134	0	0	0	134	134	73	0	0	0	0	0	0
643.	9.0	141	0	0	141	0	0	0	141	141	77	0	0	0	0	0	0
745.	10.0	142	0	0	163	0	0	0	163	117	157	0	0	0	0	0	0
846.	11.0	144	0	0	180	50	0	0	169	143	157	0	0	0	0	0	0
908.	12.0	140	0	0	180	91	0	0	169	168	157	0	0	0	0	0	0
967.	13.0	162	0	0	180	50	78	0	169	168	157	0	0	0	0	0	0
953.	14.0	162	0	0	180	50	63	0	169	168	157	0	0	0	0	0	0
881.	15.0	140	0	0	180	0	65	0	169	168	157	0	0	0	0	0	0
869.	16.0	140	0	0	180	0	52	0	169	168	157	0	0	0	0	0	0
934.	17.0	163	0	0	180	50	50	0	169	162	157	0	0	0	0	0	0
997.	18.0	161	0	0	180	58	100	0	169	168	157	0	0	0	0	0	0
976.	19.0	162	0	0	180	50	87	0	169	168	157	0	0	0	0	0	0
920.	20.0	141	0	0	175	102	0	0	174	168	157	0	0	0	0	0	0
881.	21.0	141	0	0	180	63	0	0	169	168	157	0	0	0	0	0	0
864.	22.0	142	0	0	180	50	0	0	169	164	157	0	0	0	0	0	0
869.	23.0	141	0	0	180	51	0	0	169	168	157	0	0	0	0	0	0
869.	24.0	141	0	0	180	50	0	0	169	168	157	0	0	0	0	0	0

Figure 6.5 'Ideal' Prediction

Run time 0 h 4 min 23 s

Reserve cover = 0.68

Fuel cost (£) = 204851.2

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
876.	1.0	141	0	0	180	58	0	0	169	168	157	0	0	0	0	0	0
845.	2.0	144	0	0	180	50	0	0	169	141	157	0	0	0	0	0	0
783.	3.0	132	0	0	168	0	0	0	169	162	149	0	0	0	0	0	0
751.	4.0	140	0	0	165	0	0	0	165	122	157	0	0	0	0	0	0
717.	5.0	151	0	0	157	0	0	0	157	91	157	0	0	0	0	0	0
670.	6.0	147	0	0	147	0	0	0	147	147	80	0	0	0	0	0	0
645.	7.0	141	0	0	141	0	0	0	141	141	77	0	0	0	0	0	0
632.	8.0	138	0	0	138	0	0	0	138	138	75	0	0	0	0	0	0
665.	9.0	146	0	0	146	0	0	0	146	146	79	0	0	0	0	0	0
767.	10.0	134	0	0	168	0	0	0	168	144	151	0	0	0	0	0	0
864.	11.0	142	0	0	180	50	0	0	169	164	157	0	0	0	0	0	0
929.	12.0	164	0	0	180	50	**50**	0	169	156	157	0	0	0	0	0	0
991.	13.0	161	0	0	180	52	100	0	169	168	157	0	0	0	0	0	0
965.	14.0	162	0	0	180	50	76	0	169	168	157	0	0	0	0	0	0
879.	15.0	141	0	0	180	**61**	0	0	169	168	157	0	0	0	0	0	0
862.	16.0	142	0	0	180	**50**	0	0	169	161	157	0	0	0	0	0	0
956.	17.0	162	0	0	180	50	67	0	169	168	157	0	0	0	0	0	0
1041.	18.0	149	0	0	180	102	100	0	181	168	157	0	0	0	0	0	0
1014.	19.0	161	0	0	180	75	100	0	169	168	157	0	0	0	0	0	0
959.	20.0	162	0	0	180	50	**70**	0	169	168	157	0	0	0	0	0	0
920.	21.0	165	0	0	180	50	**50**	0	169	146	157	0	0	0	0	0	0
970.	22.0	162	0	0	180	50	**81**	0	169	168	157	0	0	0	0	0	0
894.	23.0	140	0	0	180	76	0	0	169	168	157	0	0	0	0	0	0
860.	24.0	142	0	0	180	50	0	0	169	160	157	0	0	0	0	0	0

Figure 6.6 Univariate Unit Commitment Schedule

Reserve cover = 0.68

Fuel cost (£) = 199385.4

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
870.	1.0	141	0	0	180	51	0	0	169	168	157	0	0	0	0	0	0
828.	2.0	147	0	0	180	50	0	0	169	122	157	0	0	0	0	0	0
756.	3.0	138	0	0	166	0	0	0	166	130	155	0	0	0	0	0	0
722.	4.0	150	0	0	158	0	0	0	158	96	157	0	0	0	0	0	0
693.	5.0	152	0	0	152	0	0	0	152	109	126	0	0	0	0	0	0
648.	6.0	142	0	0	142	0	0	0	142	142	77	0	0	0	0	0	0
625.	7.0	137	0	0	137	0	0	0	137	137	75	0	0	0	0	0	0
613.	8.0	134	0	0	134	0	0	0	134	134	73	0	0	0	0	0	0
643.	9.0	141	0	0	141	0	0	0	141	141	77	0	0	0	0	0	0
745.	10.0	142	0	0	163	0	0	0	163	117	157	0	0	0	0	0	0
846.	11.0	144	0	0	180	50	0	0	169	143	157	0	0	0	0	0	0
908.	12.0	167	0	0	180	50	50	0	169	132	157	0	0	0	0	0	0
967.	13.0	162	0	0	180	50	78	0	169	168	157	0	0	0	0	0	0
953.	14.0	162	0	0	180	50	63	0	169	168	157	0	0	0	0	0	0
881.	15.0	141	0	0	180	63	0	0	169	168	157	0	0	0	0	0	0
869.	16.0	141	0	0	180	50	0	0	169	168	157	0	0	0	0	0	0
934.	17.0	163	0	0	180	50	50	0	169	162	157	0	0	0	0	0	0
997.	18.0	161	0	0	180	58	100	0	169	168	157	0	0	0	0	0	0
976.	19.0	162	0	0	180	50	87	0	169	168	157	0	0	0	0	0	0
920.	20.0	165	0	0	180	50	50	0	169	145	157	0	0	0	0	0	0
881.	21.0	171	0	0	180	50	50	0	169	101	157	0	0	0	0	0	0
864.	22.0	173	0	0	180	50	50	0	169	82	157	0	0	0	0	0	0
869.	23.0	141	0	0	180	51	0	0	169	168	157	0	0	0	0	0	0
869.	24.0	141	0	0	180	50	0	0	169	168	157	0	0	0	0	0	0

Figure 6.7 Univariate Unit Commitment Operation

c. Multivariate Prediction (MAPE : 1.44%)

i. Forecasted Unit Commitment Schedule

Again, a 24-point loading calculation was effected but with the 24h ahead multivariate forecasted demand data, Figure 6.8. Thus the multivariate forecasted unit commitments were obtained.

ii. Actual Demand Profile with Forecasted Unit Commitments

Another loading calculation was then carried out using the actual demand data and the multivariate unit commitments with unit availability controlled as before, see Firure 6.9.

Fuel Cost = £199,190

Forecasting Deficit = £199,190 - £199,125

= £65

This is only a 0.03% increase from ideal prediction.

Run time 0 h 4 min 12 s

Reserve cover = 0.68

Fuel cost (£) = 200012.7

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
875.	1.0	141	0	0	180	57	0	0	169	168	157	0	0	0	0	0	0
841.	2.0	145	0	0	180	50	0	0	169	137	157	0	0	0	0	0	0
774.	3.0	131	0	0	170	0	0	0	169	153	148	0	0	0	0	0	0
737.	4.0	144	0	0	162	0	0	0	162	110	157	0	0	0	0	0	0
703.	5.0	153	0	0	154	0	0	0	154	85	154	0	0	0	0	0	0
653.	6.0	143	0	0	143	0	0	0	143	143	78	0	0	0	0	0	0
624.	7.0	137	0	0	137	0	0	0	137	137	74	0	0	0	0	0	0
608.	8.0	133	0	0	133	0	0	0	133	133	72	0	0	0	0	0	0
641.	9.0	141	0	0	141	0	0	0	141	141	76	0	0	0	0	0	0
745.	10.0	142	0	0	163	0	0	0	163	116	157	0	0	0	0	0	0
842.	11.0	145	0	0	180	50	0	0	169	138	157	0	0	0	0	0	0
907.	12.0	140	0	0	180	90	0	0	169	168	157	0	0	0	0	0	0
972.	13.0	162	0	0	180	50	83	0	169	168	157	0	0	0	0	0	0
950.	14.0	162	0	0	180	50	60	0	169	168	157	0	0	0	0	0	0
867.	15.0	140	0	0	180	0	50	0	169	168	157	0	0	0	0	0	0
854.	16.0	142	0	0	180	0	50	0	169	153	157	0	0	0	0	0	0
942.	17.0	162	0	0	180	50	52	0	169	168	157	0	0	0	0	0	0
1019.	18.0	161	0	0	180	80	100	0	169	168	157	0	0	0	0	0	0
991.	19.0	161	0	0	180	52	100	0	169	168	157	0	0	0	0	0	0
936.	20.0	163	0	0	180	50	50	0	169	164	157	0	0	0	0	0	0
896.	21.0	140	0	0	180	78	0	0	169	168	157	0	0	0	0	0	0
882.	22.0	141	0	0	180	64	0	0	169	168	157	0	0	0	0	0	0
867.	23.0	141	0	0	180	50	0	0	169	167	157	0	0	0	0	0	0
835.	24.0	146	0	0	180	50	0	0	169	131	157	0	0	0	0	0	0

Figure 6.8 Multivariate Unit Commitment Schedule

Reserve cover = 0.68

Fuel cost (£) = 199190.3

Demand (MW)	Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
870.	1.0	141	0	0	180	51	0	0	169	168	157	0	0	0	0	0	0
828.	2.0	147	0	0	180	50	0	0	169	122	157	0	0	0	0	0	0
756.	3.0	138	0	0	166	0	0	0	166	130	155	0	0	0	0	0	0
722.	4.0	150	0	0	158	0	0	0	158	96	157	0	0	0	0	0	0
693.	5.0	152	0	0	152	0	0	0	152	109	126	0	0	0	0	0	0
648.	6.0	142	0	0	142	0	0	0	142	142	77	0	0	0	0	0	0
625.	7.0	137	0	0	137	0	0	0	137	137	75	0	0	0	0	0	0
613.	8.0	134	0	0	134	0	0	0	134	134	73	0	0	0	0	0	0
643.	9.0	141	0	0	141	0	0	0	141	141	77	0	0	0	0	0	0
745.	10.0	142	0	0	163	0	0	0	163	117	157	0	0	0	0	0	0
846.	11.0	144	0	0	180	50	0	0	169	143	157	0	0	0	0	0	0
908.	12.0	140	0	0	180	91	0	0	169	168	157	0	0	0	0	0	0
967.	13.0	162	0	0	180	50	78	0	169	168	157	0	0	0	0	0	0
953.	14.0	162	0	0	180	50	63	0	169	168	157	0	0	0	0	0	0
881.	15.0	140	0	0	180	0	65	0	169	168	157	0	0	0	0	0	0
869.	16.0	140	0	0	180	0	52	0	169	168	157	0	0	0	0	0	0
934.	17.0	163	0	0	180	50	50	0	169	162	157	0	0	0	0	0	0
997.	18.0	161	0	0	180	58	100	0	169	168	157	0	0	0	0	0	0
976.	19.0	162	0	0	180	50	87	0	169	168	157	0	0	0	0	0	0
920.	20.0	141	0	0	180	50	50	0	169	145	157	0	0	0	0	0	0
881.	21.0	141	0	0	180	63	0	0	169	168	157	0	0	0	0	0	0
864.	22.0	142	0	0	180	50	0	0	169	164	157	0	0	0	0	0	0
869.	23.0	141	0	0	180	51	0	0	169	168	157	0	0	0	0	0	0
869.	24.0	141	0	0	180	50	0	0	169	168	157	0	0	0	0	0	0

Figure 6.9 Multivariate Unit Commitment Operation

The above example indicates that employing multivariate forecasting can reduce the forecast deficit arising from prediction error for 24h ahead. This example may exaggerate the improvement since a Sunday is being considered and demand is light, hence all major generating units may not be committed and consequently these units may display increased volatility.

The above analysis also assumes that the economic loading calculation, and the forecast on which it is based, is performed once only, at the beginning of a 24h period. In practice, both forecasting and loading programs would probably be run more frequently, at most every 2h, when the univariate method is capable of securing practically full forecasting credit.

Generally, the ability of the loading algorithm to determine correct start-up times with reasonable accuracy may be due to the manner in which these errors are distributed, with the greatest errors (in terms of both magnitude and implication) occurring at peak times. This would be a problem in generation planning, but can be tolerated to some extent in generation scheduling, for the following reason. When demand is high, most major available units are already committed. Errors at these demands will therefore influence mainly the commitment of smaller units, which tend to have shorter lead times and hence can be run up and synchronised quickly.

6.7 Implicit Prediction

In Northern Ireland, wind direction exhibits a south-westerly dominance. This pattern in the wind regime may have implications for demand prediction.

The months of November through to March is the period in which strong winds are most frequent. Monitored meteorological data is available for November 1991 to March 1992. Using this data a 'wind-rose' diagram highlighting the frequency of wind direction for different groups of wind speeds was constructed, Figure 6.10. The daily average wind speed and direction was considered. This confirms the concept of a strong south-westerly trend in wind direction.

With such directional dominance, meteorological conditions near peripheral regions to windward may be surmised to anticipate conditions at the major load centre in the north-east of the province. For example, downwind demand variations in south-west provincial towns such as Enniskillen or Strabane could provide an 'early warning' for the Belfast region. Indeed, these areas have little industrial base so a considerable proportion of the demand variation prevalent here may be weather sensitive. This 'implicit' prediction could provide a novel solution to the forecasting problem within the NIE system.

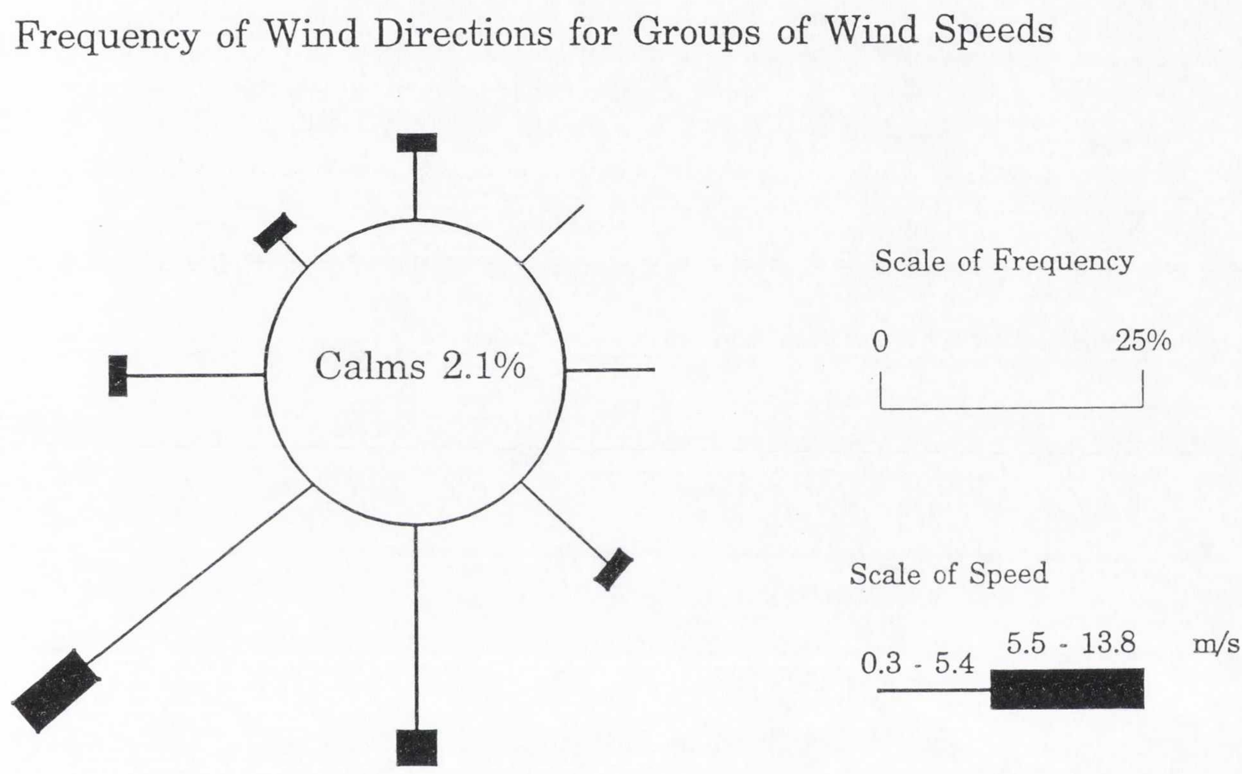


Figure 6.10 Wind-Rose Diagram

6.8 Conclusions

Multivariate time series models were developed to relate total system demand to meteorological variables. An increase in accuracy over the univariate time series method was observed. However, the improvement was not totally convincing.

Only temperature related variables proved to be of significance in the multivariate modelling process. In the case of wind speed, strong winds undoubtedly influence demand but the overall random variation of wind speed creates difficulty in time series modelling. Satisfactory inclusion of illumination in this modelling process may require a determination of the outdoor threshold illumination level - the light level at which people switch lights on. Also pressure may prove to be a more significant variable over the longer term.

The inclusion of these other weather variables in the modelling process may require the separation of the system load into weather sensitive and non-weather sensitive portions. However, this isolation of the weather dependent load is more suited to time-of-day or static models.

The Box-Jenkins method (univariate) is a dynamic load model where load behaviour is modelled as a dynamic process where the load at any one time depends not only on the time of day but on the past behaviour of the load. So, to some extent univariate modelling includes the effect of weather variations, since greater attention is given to more recent demand values, and it may be assumed that the weather will remain constant over the short-term. This may explain why multivariate modelling struggles to improve upon univariate forecasts.

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7.1 Introduction

As electricity costs increase, utilities are extending their activities to the customer's side of the meter. They are striving to control, both directly and indirectly, when and how electricity is used, thus placing an increased emphasis on demand side technologies. This 'control' is called load management and benefits utilities by allowing more uniform use of generating equipment.

According to Gellings (1) 'the most effective tool of load management is load control, in which consumers' equipment is turned on or off to change the total demand on the utility'. Time switching is one major load control scheme in which electric loads are simply turned on or off for a given period on the basis of time-of-day rates. Within the NIE system, loads such as space and water heating, which can readily be deferred to off-peak hours, are controlled by time switching. The storage of energy for heating during off-peak hours for use during peak hours is acceptable to consumers since it requires no change in their usage patterns. Indeed, in the recent past many authors (2-7) have considered the direct control of water and space heating loads for demand side management.

This chapter describes measurements taken to determine the profile of a typical off-peak load. An evaluation of the benefit of using an interruptible system off-peak load with this profile for load shedding for emergency reserve substitution is then carried out for the NIE system. Finally, the practical implementation of a distributed load shedding scheme is considered.

7.2 Off-Peak Load as Reserve

7.2.1 Introduction

It is a fundamental requirement of power system operation that sufficient emergency reserve should be scheduled to cover the deficit arising from the loss of a major generating unit. This is a particularly taxing requirement in the case of systems where the largest infeed exceeds 10% of the demand and where there are no interconnections with other systems.

The emergence of a controlled component of load, e.g. 'Economy 7', offers the possibility of an interruptible load at certain times. Most Economy 7 load is such that an interruption in supply of a few minutes causes little or no inconvenience. However the benefit to the system of this benign load shedding is considerable (8).

Measurements were taken using a simple data acquisition system to determine the profile of a typical off-peak load. The benefit of an interruptible system off-peak load with this profile was then evaluated for the NIE system.

7.2.2 Off-peak Monitoring

An Economy 7 installation of 22kW per phase was monitored for a period of some months. The load is time switched to provide a charging period of 7 hours commencing at midnight. Thermostats within each heater also regulate the energy input during the 7 hour period.

To enable this load to be monitored non-invasively a current transformer (CT) was placed on one phase of the main supply cable of the installation. The sinusoidal output of the CT was rectified to provide a direct voltage representation of load current. This direct voltage was recorded at five minute intervals by a simple data acquisition system comprising a PC and associated 12-bit analogue-to-digital interface. The interface (9) has a capability of 16 channels of single-ended analogue inputs. A 50 way insulation displacement connector links the input channels to a 50 way screw terminal box. A program in BASIC was used to read specified channels, corresponding to each transducer, convert the voltage to the relevant quantity and store the data on floppy disk.

Tests for individual periods indicate that for a short time period all heaters are on and full load current is drawn. The duration of this time period varies with the season of the year. The time period is longest for winter, Figure 7.4, and shortest for summer, Figure 7.2. The individual profiles for spring and autumn are shown in Figures 7.1 and 7.3 respectively. In all cases when this period of full load current has expired thermal cycling of individual heaters occurs. It is also apparent that the total nightly energy consumption (area under curve) varies significantly between seasons. Current averaged over a series of nights indicates a smoother decrease of power consumption from full load, Figure 7.5.

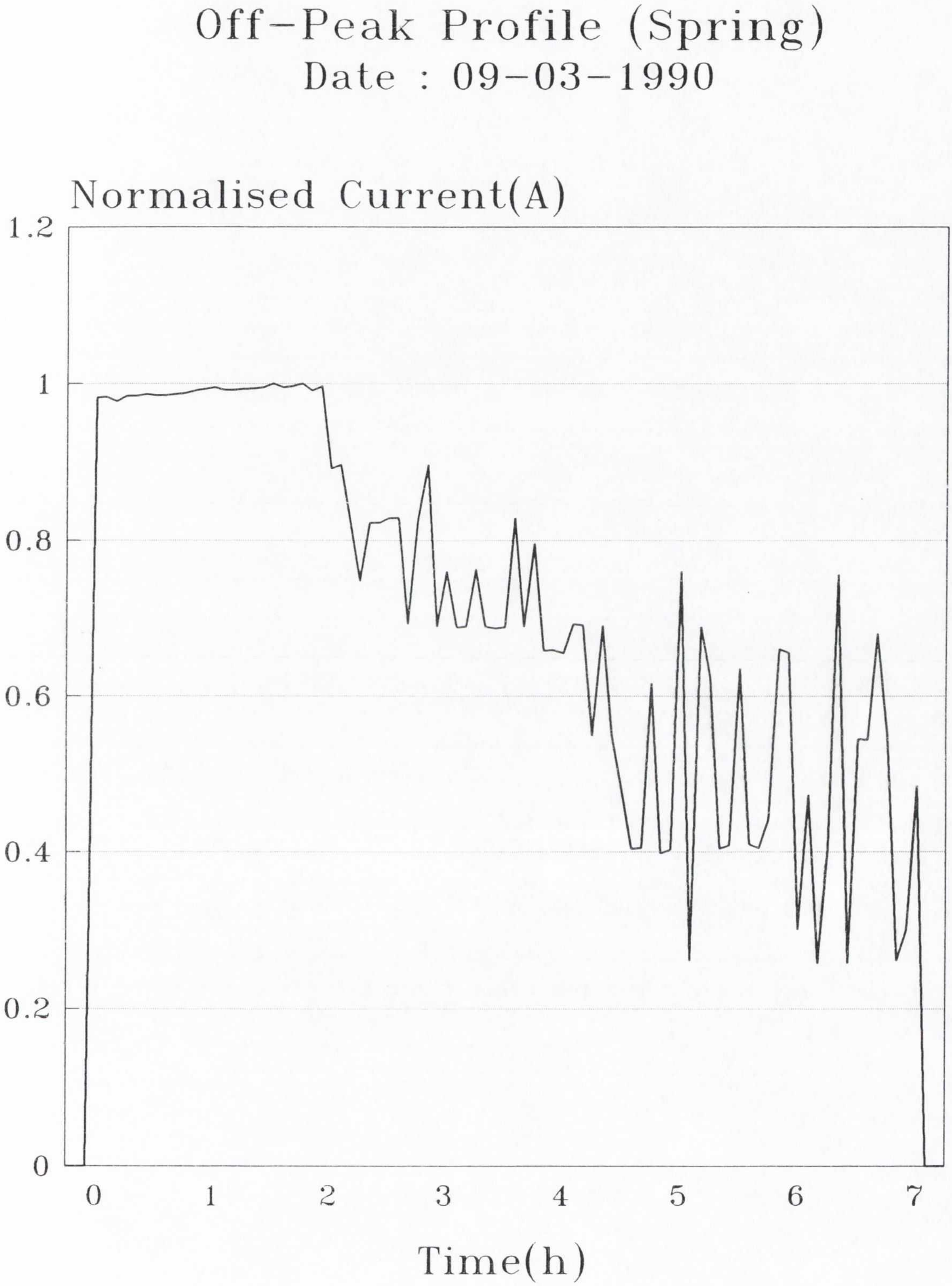

Figure 7.1 Off-Peak Load Variation (Spring)

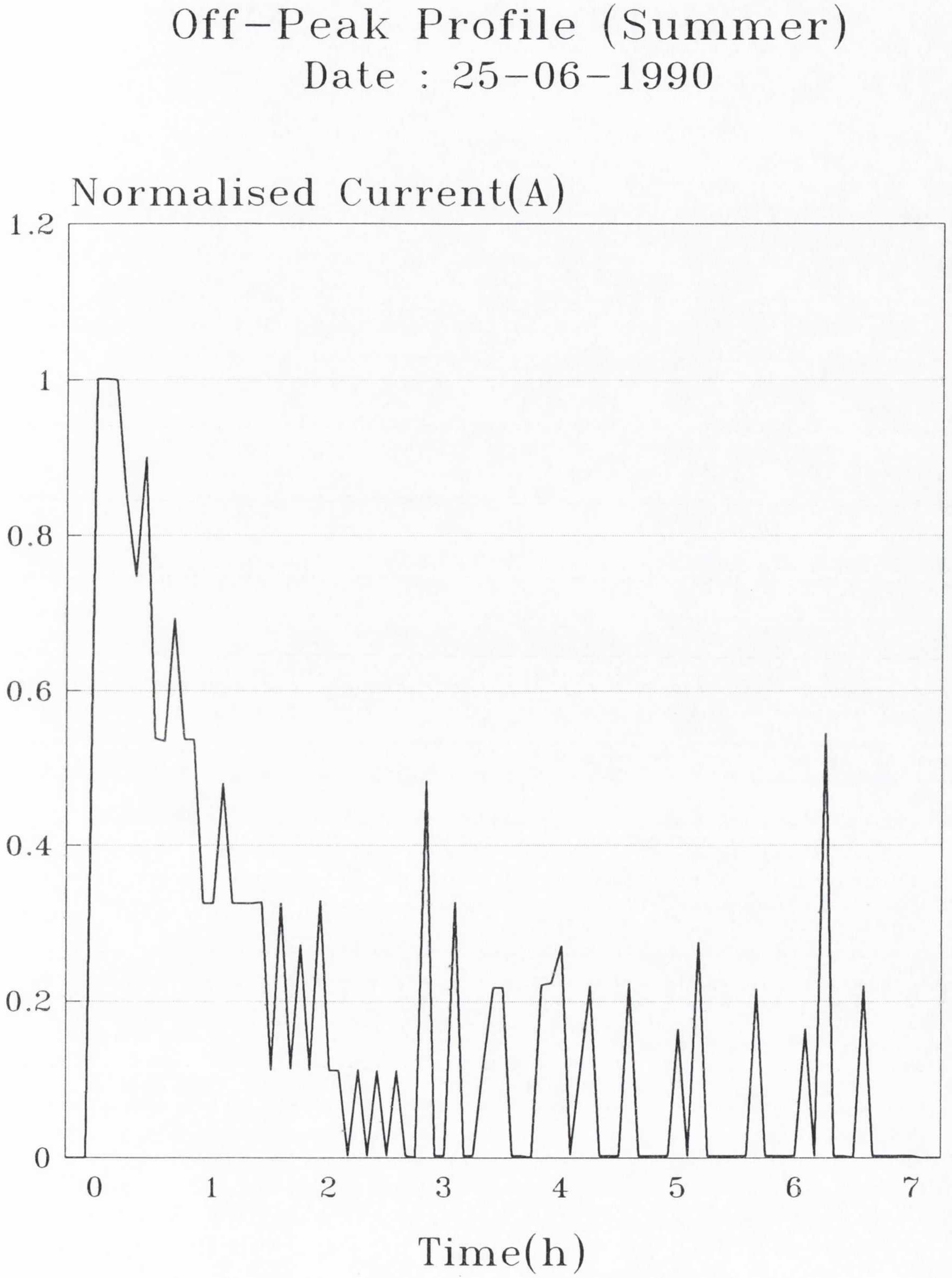

Figure 7.2 Off-Peak Load Variation (Summer)

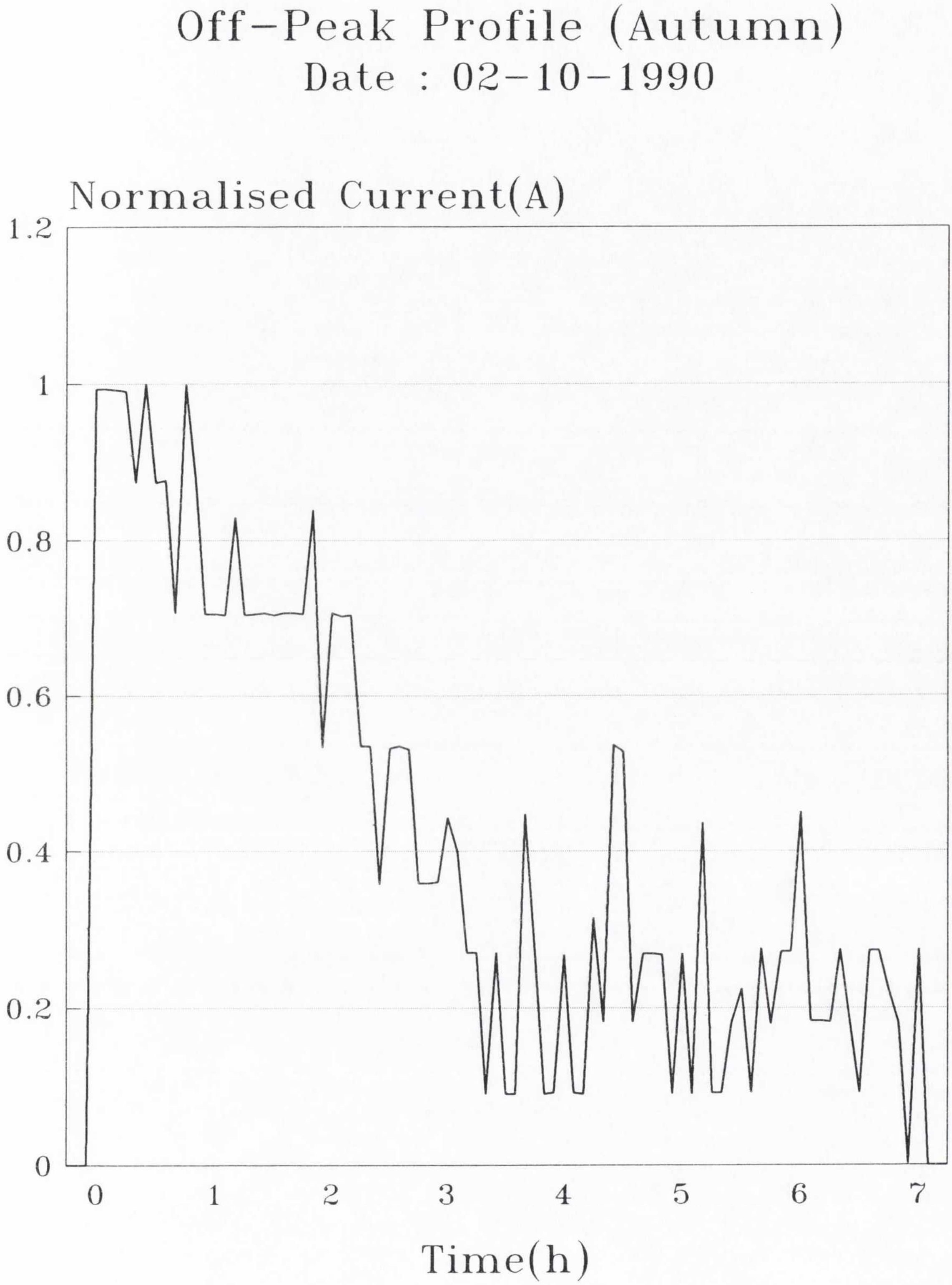

Figure 7.3 Off-Peak Load Variation (Autumn)

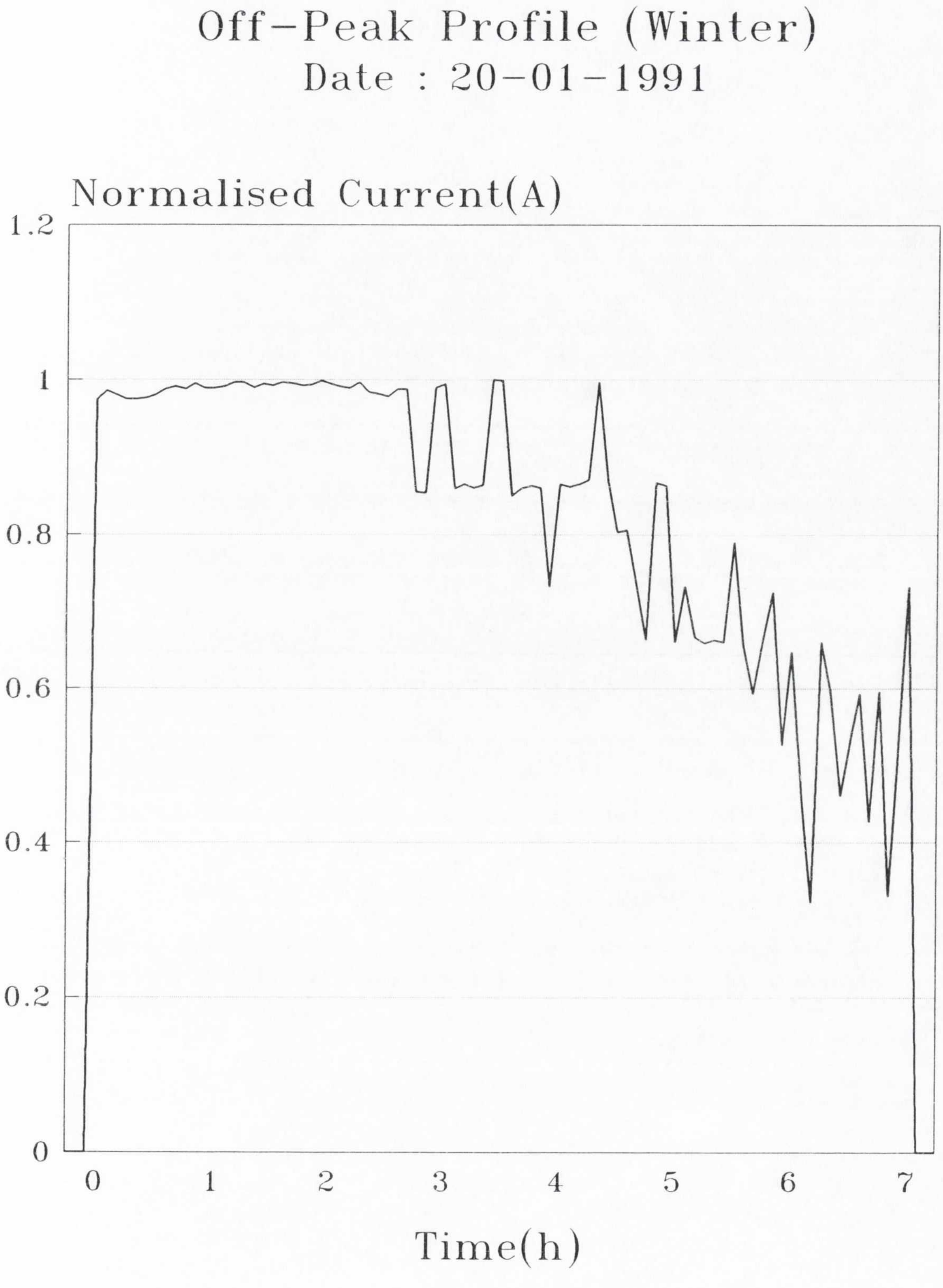

Figure 7.4 Off-Peak Load Variation (Winter)

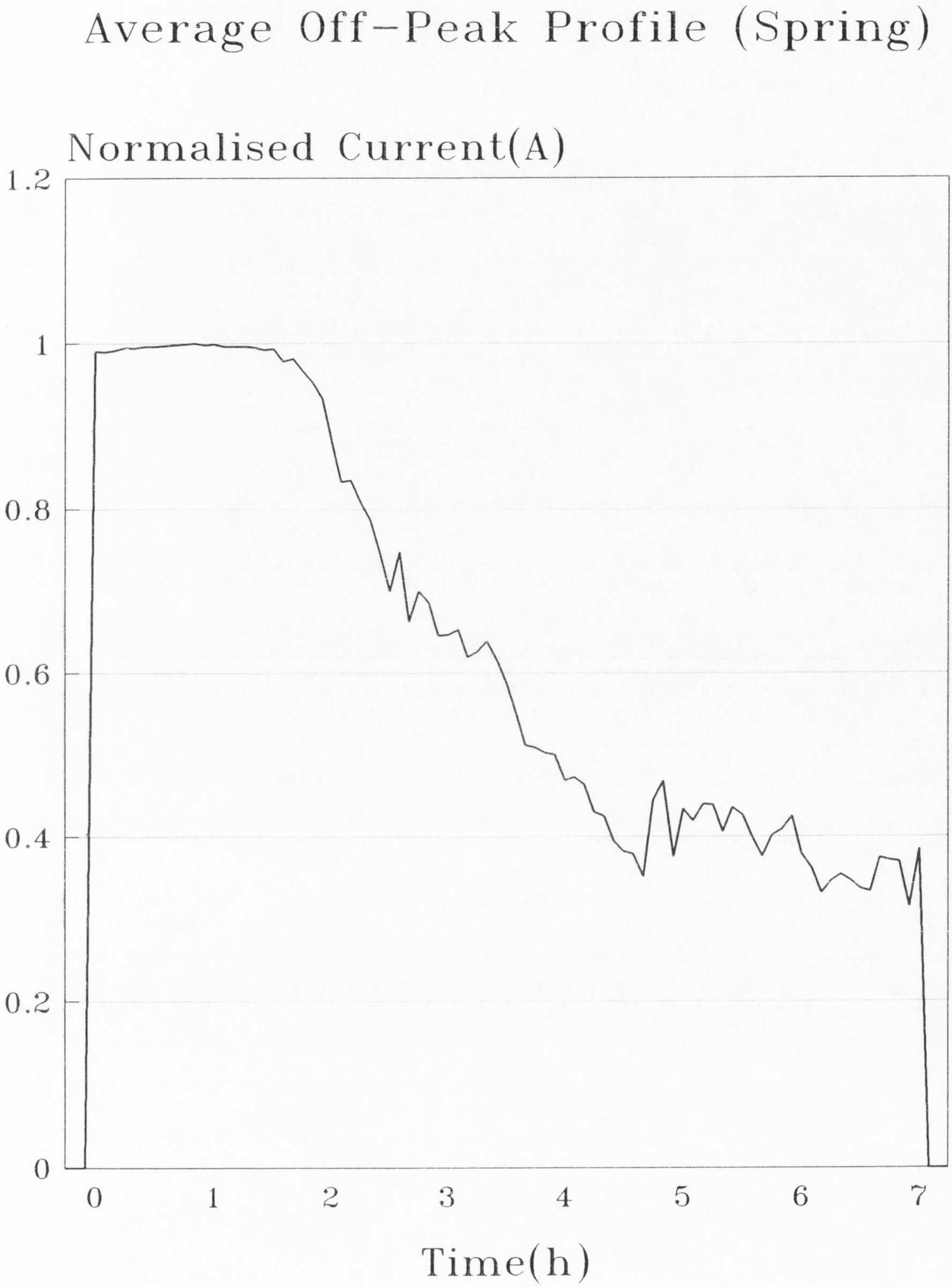

Figure 7.5 Average Off-Peak Load Variation

7.2.3 Economic Assessment

In order to assess the benefit of interruptible off-peak load, a 24-h economic scheduling calculation was performed for the NIE system. As in previous analysis (Chapter One and Chapter Three), thirteen units were employed, with four 'must-run' units (1,4,9,10) being used. The reserve and other unit parameters are detailed in Appendix 1.

As outlined previously, the economic loading program minimises the summated unit costs, subject to the sum of the unit outputs equalling the demand, and subject also to security constraints. These are designed to ensure that loss of a plant item, such as a generating unit or a transmission line, can be sustained without an unacceptable level of consumer disconnection.

The main concern here is with possible generation loss in an isolated supply system. A system reserve constraint was therefore included in the loading program to ensure that a required proportion of each generating unit infeed is covered by emergency reserve from remaining units. In the present work, interruptible load is considered as additional emergency reserve.

Economic loading over a 24-h period was considered. A typical demand variation for a spring day is given in Table 7.1 and shown in Figure 7.6. The total amount of off-peak energy was taken to be 5.93% of the total, in line with annual off-peak energy consumption for 1989/90, as indicated by NIE system operations staff. This, taken together with the demand profile based on the measurements, yields the system off-peak variation given in Table 7.1, and shown in Figure 7.6.

Loading calculations were performed initially without crediting the off-peak load with reserve capability. The calculations were then repeated with the off-peak load included in the emergency reserve. In both cases reserve was scheduled to cover 68% of the potential infeed loss. The generation costs were, Non-interruptible off-peak load: £203,896

Interruptible off-peak load: £202,375

This is a saving of £1,521 over 24-h, or 0.75%.

Hour	Demand(MW)	Off-peak(MW)
1	705	249
2	702	242
3	716	184
4	711	140
5	677	104
6	655	103
7	692	88
8	830	0
9	971	0
10	1026	0
11	1029	0
12	1023	0
13	1010	0
14	959	0
15	925	0
16	928	0
17	967	0
18	977	0
19	890	0
20	813	0
21	847	0
22	898	0
23	853	0
24	719	0

Table 7.1 - Demand Profile

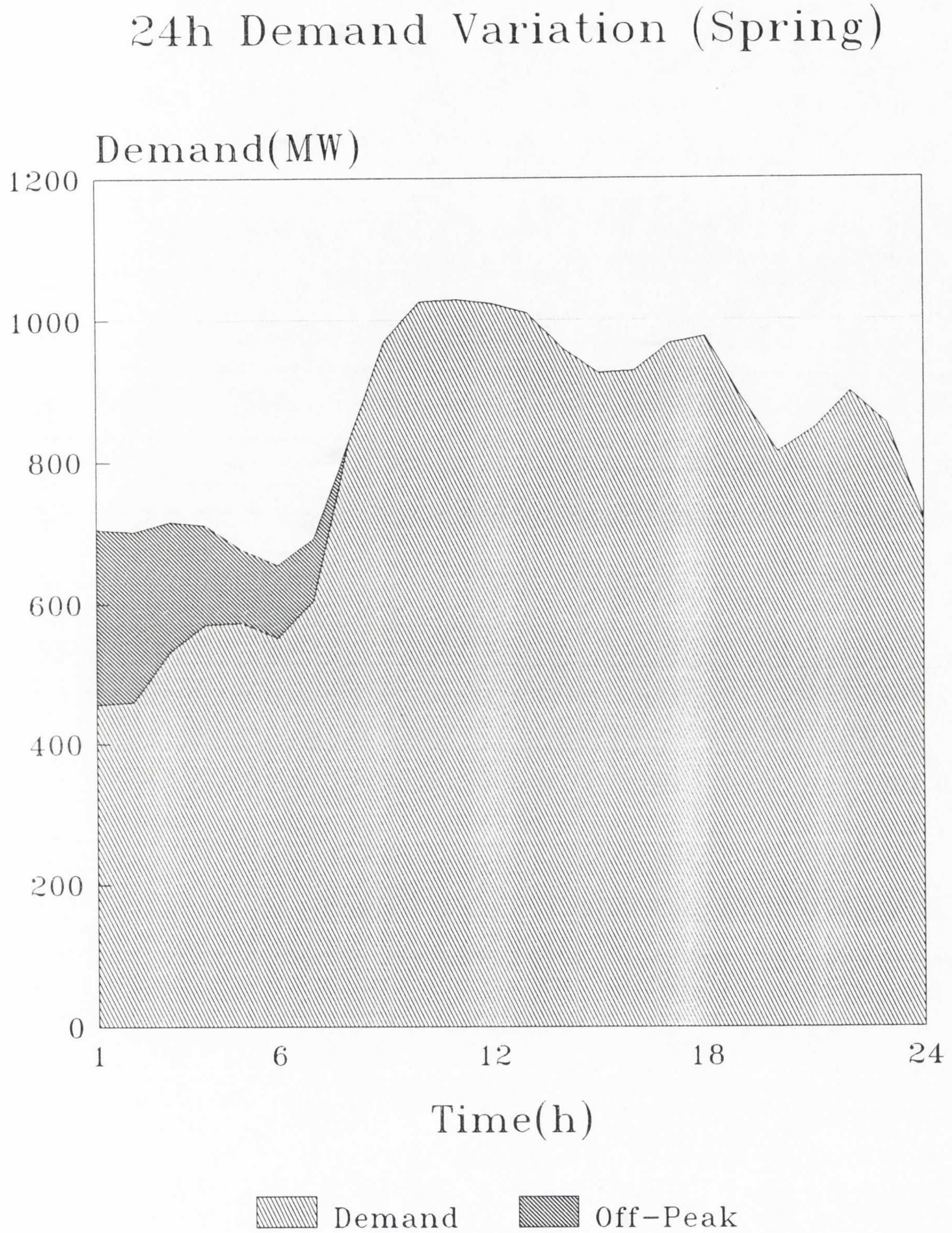

Figure 7.6 System 24-h Demand Variation

7.2.4 Off-Peak Load Forecasting

The interruptible load which may be available through temporary disconnection of off-peak supply can only replace conventional reserve if the magnitude of the appropriate load is known. It is therefore important to forecast this component of load since it is not feasible to measure it. However, to facilitate forecasting a knowledge of the nature of the load and past meteorological conditions (temperature) are required.

To date, off-peak heating load data has been gathered on two fronts. Firstly, direct measurements have been taken on the 66-kW installation mentioned above. Also, a radio teleswitching facility has been used to disconnect off-peak load for short periods on the NIE system : the relevant load may be inferred from the magnitude of the resulting impulses, of several minutes duration, on the demand curve. An example of the results thus yielded are listed in Table 7.2. Off-peak load may be estimated by relating the individual installation profile to this overall teleswitch information and then by assessing the influence of meteorological conditions. Also, correlation of total daily energy consumption at the off-peak installation with temperature may prove useful.

CODE A TELESWITCH DEMAND TESTS

Contact D - Storage Heating

16/17 Jan 90		13/14 Feb 90		13/14 Mar 90		10/11 Apr 90	
23:00	ON	23:00	ON	23:00	ON	00:00	ON
23:30	26 MW	23.28	22 MW	23:30	21 MW	00:30	24 MW
24:00	OFF	24:00	OFF	24:00	OFF	01:00	OFF
02:00	ON	02:00	ON	02:00	ON	03:00	ON
02:30	21 MW	02:27	29 MW	02:30	16 MW	03:15	21 MW
03:15	22 MW	03:15	27 MW	03:15	23 MW	04:00	16 MW
04:15	17 MW	04:15	23 MW	04:15	20 MW	04:45	13 MW
04:45	15 MW	04:45	23 MW	04:45	12 MW	05:45	9 MW
05:30	13 MW	05:30	21 MW	05:30	16 MW	06:38	10 MW
06:15	13 MW	06:13	14 MW	06:15	8 MW	07:15	9 MW
07:00	11 MW	06:59	16 MW	07:00	13 MW	08:00	7 MW
07:37		07:41	20 MW	07:30		08:45	
08:00	OFF	08:00	OFF	08:00	OFF	09:00	OFF

Contact C - Water Heating

04:00	ON	04:00	ON	04:00	ON	05:00	ON
04:30	2 MW	04:30	1 MW	04:30	TLTR	05:15	TLTR
05:00	1 MW	05:00	1 MW	05:00	TLTR	06:00	TLTR
05:45	1 MW	05:45		05:45	TLTR	06:40	TLTR
06:30	1 MW	06.30		06:30	TLTR	07:45	TLTR
09:00	OFF	08:00	OFF	08:00	OFF	09:00	

Temp	8.6 °C	1.8 °C	6.1 °C	6.9 °C
Auto Ch	6.5 hours	7.0 hours	4.5 hours	3.0 hours
No TS's	4212	4212	4285	4285
@ 31.12.89		@ 31.12.89	@ 0.04.90	@ 30.04.90

Table 7.2 Teleswitch Test Data

7.3 Under-frequency Relay for Distributed Load Shedding

7.3.1 Introduction

The previous economic analysis for the NIE system demonstrated significant fuel cost savings if conventional emergency reserve obtained by part loading thermal generating units can be replaced by the proposed interruptible load. The next step is to examine the practical aspects of using this low-priority load for distributed load-shedding (10). However in order to replace emergency reserve with controlled load, it is essential to be able to switch off the relevant load within a few seconds of generation loss.

7.3.2 Load Controllers

Supply authorities in the U.K. have been replacing the time switches traditionally used to control off-peak load with radio-controlled switches - 'radio teleswitching'. Messages are transmitted through coded phase modulation of the 200 kHz Radio 4 signal. Control messages can be transmitted in about 2 s. This system offers much more flexible control of off-peak load.

The use of radio teleswitching to disconnect off-peak load for a few minutes following sudden loss of a large generator is possible in principle. When the associated frequency fall of say 1% has been detected, a message would be transmitted causing the appropriate loads to be disconnected. However, in the current implementation, the broadcasting authority requires several minutes to assemble the control signal, so that any subsequent load relief cannot be considered to be emergency reserve.

A special purpose controller designed to disconnect off-peak load rapidly in the event of a generation loss is required. Ideally, such a device would utilise the power supply and contactors of the teleswitch. A low-cost version of the under-frequency load-shedding relays used by supply authorities world-wide to disconnect consumers during severe under-frequency transients would be highly appropriate. Such an under-frequency relay will now be outlined.

7.3.3 Under-frequency Relay

System frequency is determined by measurement of the period of each cycle, defined by zero crossings. This implementation utilises a 68HC11 single-chip microcomputer, which is extremely compact and capable of accurate measurement. In the measurement process only a sustained frequency decrease, lasting for at least 100 ms, and resulting in a fall of at least 0.25 Hz, is deemed to constitute a sufficient frequency deviation to merit load disconnection. This strategy ensures the detection of the frequency deviation caused by a generation outage but is insensitive to the normal variation of the supply frequency, typically within 0.1 Hz of the nominal value. The microcomputer continues to monitor frequency, and if recovery to a value within 0.1 Hz of the nominal value occurs, load reconnection is effected after a suitable delay. Simulated frequency responses indicate that the delay inherent in the frequency measurement algorithm does not degrade system response to a significant degree. A schematic diagram of the under-frequency relay is shown in Figure 7.7.

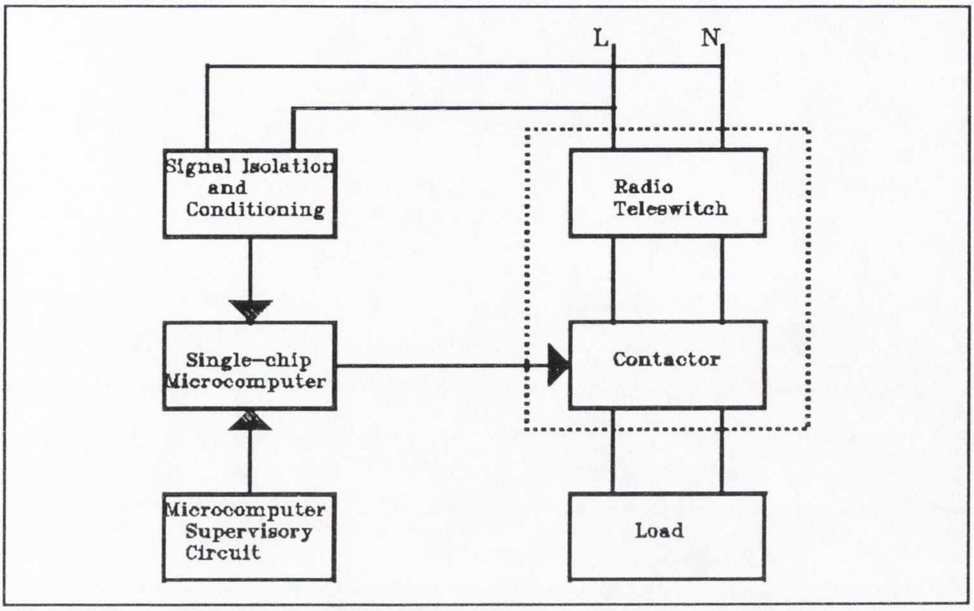

Figure 7.7 Schematic of Relay

7.3.4 Cost-benefit Analysis of Distributed Load Shedding

Generally, the cost of emergency reserve is due to two factors:

1. Extra generating capacity may have to be committed to provide the requisite amount of reserve.

2. Out-of-merit loading is usually required to provide sufficient rapid pick-up capability.

The emergency reserve requirement is particulary taxing when demand is light. At such times, the load could be supplied by a comparatively small number of large, efficient generating units with low operating costs. However the need to maintain a reliable supply is better served by utilising a larger number of smaller units, which will usually have higher fuel costs. The presence of off-peak load at such times provides an opportunity to mitigate this extra cost. The benefit of disconnecting this load, on detection of a significant fall in frequency, has been estimated for the NIE system for a spring day - see above. Taking a saving of £1,000/day over a full year, the saving for each installed kW of off-peak load (249 MW - see Table 7.1) is about £1.50 per annum.

The cost of the components of the under-frequency relay detection system described earlier is approximately £75, including power supply. Integration with a radio teleswitch would obviate the need for a separate load switch. However it is useful to consider the cost of a completely autonomous load shedding device, including the switch. The cost of the switch is load dependent, and is of the order of £3/kW. The costs and benefits of a distributed load shedding relay are shown in Figure 7.8 as a function of the installed capacity under control. The costs are based on an annual interest and depreciation charge of 20%. It may be seen that a relay integrated with a radio teleswitch, and sharing the associated load switch, is cost effective for loads of installed capacity over 10 kW. An autonomous system shows a saving for an installed capacity greater than 20 kW approximately.

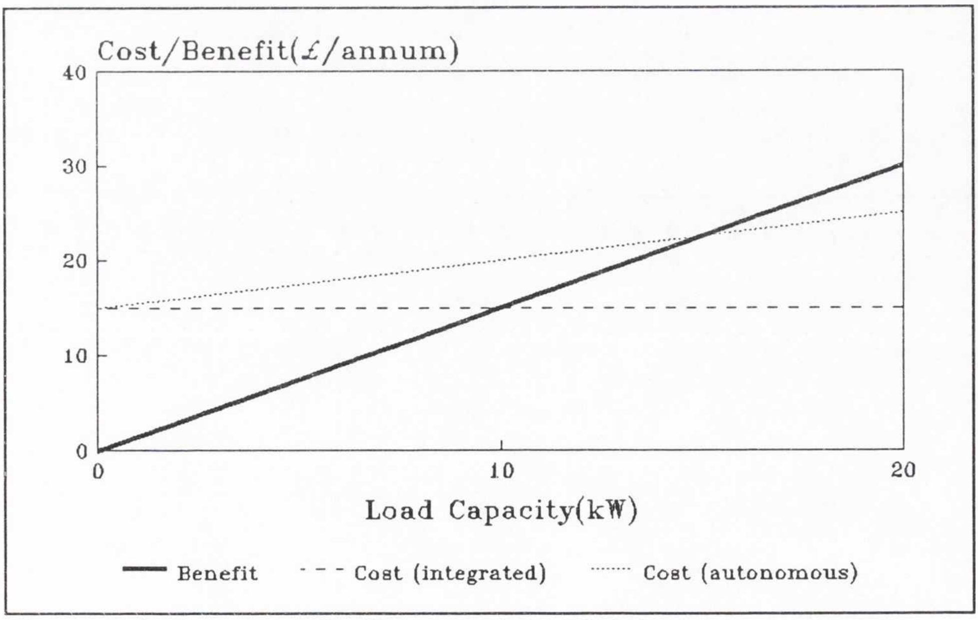

Figure 7.8 Cost-Benefit Characteristic of Relay

7.4 Conclusion

The system benefit of employing off-peak heating load for benign load shedding has been assessed through loading calculations performed on the local NIE system. The off-peak load profile was based on measurement of consumption at a particular installation. It was found that generation costs were reduced by 0.75%, or about £1.50 per installed kW of off-peak load per annum. The practical aspects of using this low priority load for distributed load shedding has also been examined.

An under-frequency load shedding relay utilising a single-chip microcomputer has been described. The device may be used to shed off-peak load, thus replacing expensive emergency reserve over the corresponding periods. The savings which accrue from the proposed approach increase with the installed capacity of the controlled load. Application to a limited number of large off-peak installations could replace a significant amount of emergency reserve in an isolated supply system at modest cost.

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8.1 Summary and Conclusion

In an isolated system, generated electricity must be dispatched to cover total consumer demand, system losses and all operating requirements of the system. At present, system demand is usually forecasted manually by dispatching engineers. This research investigated the use of a formal procedure for forecasting, incorporating the system identification techniques of univariate and multivariate Box-Jenkins time series modelling. The cost benefit accruing from each technique was also considered.

The first or introductory chapter provided background to the problem of short-term load forecasting by establishing the role of the load forecasting function in power system operation. The importance of short-term load forecasting in relation to the economic optimisation of power system operation was then determined by performing a forecasting credit evaluation, and it was subsequently shown that forecasting can, at most, reduce costs by 2%-3% over 24h. The general methodology of forecasting was outlined and a comparative review of short-term forecasting procedures carried out.

Chapter Two provided an overview of the fundamentals of Box-Jenkins univariate time series forecasting. The iterative model building procedure was outlined and relevant examples provided.

The procedures outlined in Chapter Two were employed in the course of Chapter Three to develop univariate models for the prediction of total system electrical demand. Separate models, incorporating all 7 days of the week, were developed for spring, summer and autumn. Forecast accuracy was found to be dependent on lead time used - forecast accuracy increased as lead time decreased. The forecasts produced were then used to investigate the effect of forecast accuracy on system operational costs. The prediction error distribution, i.e. the time at which the error occurred, was found to be a major influence on costs. Also, the univariate forecasting technique provided sufficient accuracy over 2h ahead to approach optimum operation when used with an economic loading algorithm, assuming that an AGC system would make the necessary on-line adjustments to synchronised generation to account for forecasting errors, and that 2h is a sufficient lead time to run up and synchronise thermal generating units. This approach was found to achieve approximately 80%-100% of the maximum cost reduction mentioned above.

The multivariate Box-Jenkins method, which is a natural extension of the univariate approach, relates independent input variables to a dependent output variable by means of transfer functions. Chapter Four summarised the theory of multivariate modelling and provided an illustrative example.

Chapter Five discussed the relationship between demand and weather, especially in relation to the incorporation of weather inputs in short-term prediction schemes. A prerequisite to formulating a weather dependent demand model is the continuous monitoring of meterological variables. So, this chapter also described a data acquisition system, capable of monitoring temperature, wind speed, light level and pressure, which has been installed at the NIE Control Centre.

Chapter Six detailed multivariate time series models which were developed to relate total system demand to weather variables. Models were developed for the winter and summer seasons using both week-day and weekend data. It was shown that an increase in accuracy over the univariate method was attainable. However, the results were not consistent. Indeed, since the univariate method takes into account recent demand values, it tends to inherently include the influence of meteorological variables. This conclusion is based upon the assumption that weather conditions remain constant over the short-term. A small reduction in operating cost is possible when using the multivariate rather than the univariate technique, corresponding to 5% of the maximum potential cost reduction.

Chapter Seven considered the direct control of water and space heating loads for demand side management. The system benefit of employing this off-peak load as a replacement for spinning reserve via benign load shedding was assessed through loading calculations performed on the NIE system. The off-peak load profile was based on measurements at a particular installation. It was found that costs were reduced by 0.75%. However, the interruptible load which may be available through temporary disconnection of off-peak supply can only replace conventional reserve if the magnitude of the appropriate load is known. Thus, the possibility of forecasting this component of load was discussed.

An under-frequency load shedding relay utilising a single-chip microcomputer, which may be used to shed off-peak load, was described also. The savings which accrue from the proposed approach increase with the installed capacity of the controlled load.

8.2 Future Work

Future work could include on-line operation of the Box-Jenkins time series method. However, implementation of the heuristic processes of model building is impractical on-line and the model to be used would have to be chosen off-line, prior to any prediction, by a modeller using a computer interactively as an aid. Subsequent on-line revisions to the model to track changes in trends and patterns in the data would be limited to re-estimating the model parameters periodically to fit new demand data as they become available. Demand data from the previous three or four weeks could be used in the estimation of the model parameters.

The statistical/mathematical approach to load prediction is usually accurate in predicting normal days, but will generally not produce very good predictions for special days (e.g. bank holidays) and weather influenced demand changes which can be non-uniform, i.e, based on human response. For example, if weather conditions do not change between consecutive weeks, then a model with a dominant weekly term provides a good forecast. However, if weather conditions are changeable, then a stronger daily term is needed because the previous day's profile will provide the best indication of demand variation.

This difficulty can be circumvented by allowing operator modification and manipulation of data, by including libraries of special day curves etc.. Alternatively, this approach may be realised in the future by application of expert system or knowledge-based engineering technology. In essence these are software systems based on knowledge rather than inference. The aim is to combine the mathematical approach with the ability to emulate, to some extent, the knowledge and reasoning of an experienced operator.

Wind direction, which manifests a south-westerly dominance in the province, may provide a novel solution to the forecasting problem. The feasibility of an implicit forecasting scheme, in which downwind demand variations in the north-east of the province might mirror earlier variations in south-western areas, is worthy of further investigation.

Consideration of the direct control of off-peak water and space heating loads has shown that employing this off-peak load for benign load shedding could replace a significant amount of emergency reserve at modest cost. A requirement of this scheme is the ability to forecast this component of load - an area where future work needs to be carried out. Other loads with high thermal lags, e.g. refrigeration load, may be available for temporary disconnection, and hence have potential to replace conventional reserve. Again, there is a need to forecast this component of load.

Expanding opportunities, and enhanced technologies, for load management are leading to a requirement for disaggregate load forecasting. The information thus gained would have implications beyond the operational time-scale considered here, and would influence future electrical energy marketing and tariff strategy.

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A description of the economic loading algorithm employed in this research is provided here. The economic scheduling process consists of two parts : dynamic programming and economic dispatch. Dynamic programming finds the optimum path, between the start of the problem and the end, from a number of possible unit commitment solutions. Economic dispatch determines the optimum unit outputs required to meet a demand subject to the various constraints. These two elements will be discussed in the following sections.

A1.1 Dynamic Programming Process

Dynamic programming is a methodical procedure which systematically evaluates a large number of possible decisions in a multi-step problem. The dynamic scheduling problem is by its nature complex. This is due to the fact that if there are N generating units to be scheduled, then over P periods there are 2^{NP} combinations to be checked, costed, and compared with each other. The dynamic programming method produces a unit commitment schedule and unit outputs so that the overall cost is a minimum. The dynamic commitment and unit loading process implemented here is outlined in Figure A1.1. The usual scheduling problem may be formulated over multiple scheduling intervals as:

$$Z=\sum_{i=1}^{N}\sum_{j=1}^{P}[\ \lambda_i^j\ a_i\ T_j\ +\ b_i\ x_i^j\ T_j\ +\ \phi_i^j\ S_i^j\ ] \tag{A1.1}$$

where

N = number of generating units

P = number of scheduling intervals

T_j = time interval j

Z = operating cost over P intervals

a_i = unit i no-load cost

b_i = unit i incremental cost

x^i_j = unit i output during T_j

λ_i^j = 1 if unit i 'on' during T_j, otherwise 0

ϕ_i^j = 1 if unit i starts during T_j, otherwise 0

S_i^j = start-up cost of unit i at the beginning of interval T_j

The minimisation is subject to demand and unit loading constraints:

$$\sum_{i=0}^{N}\lambda_i^j\ x_i^j\ \geq\ D_j \tag{A1.2}$$

where D_j is the demand during T_j and:

$$\lambda_i^j\ l_i\ \leq\ x_i^j\ \leq\ \lambda_i^j\ u_i\quad (all\ j) \tag{A1.3}$$

where l_i and u_i are the lower and upper limits of unit i respectively. To ensure security of supply following possible generator outage, emergency reserve constraints are included. Conventionally, these are formulated to ensure that a proportion R, where $0 \leq R \leq 1$, of each infeed x^j_i is covered by reserve, r, from the remaining units.

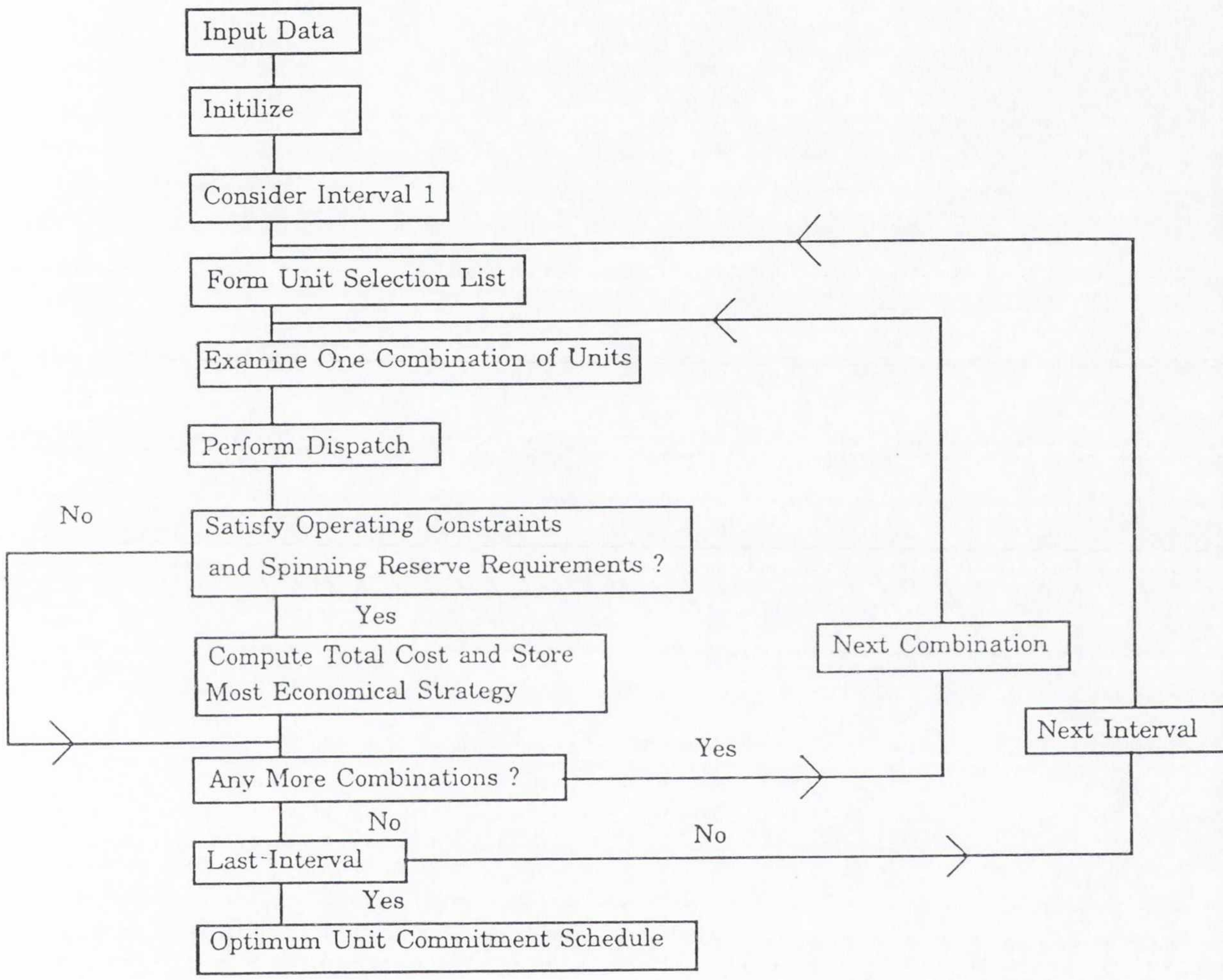

Figure A1.1 Dynamic Unit Commitment Program

Due to the large size of the scheduling problem, modern scheduling algorithms concentrate on elimination of as many combinations as possible to avoid performing calculations for those combinations. The solution process is initiated by decoupling the scheduling points, reducing the size of the problem from 2^{NP} to $P*2^N$ combinations. The feasible unit combinations of each scheduling point are identified by an efficient search process which prunes the search tree on the basis of generating capacity limitation. The dynamic programming algorithm used here works forward in time, and stores a single backward path to the first scheduling point, the retained path being the one with the lowest accumulated cost.

The commitment process uses a constraint-directed approach at each scheduling interval. The commitment variables assume only the values '0' for uncommitted and '1' for committed units. Initially, all units are considered to be committed. Part of a constraint-directed search tree is shown in Figure A1.2, in which shaded nodes are repeats of earlier combinations and do not therefore require further examination.

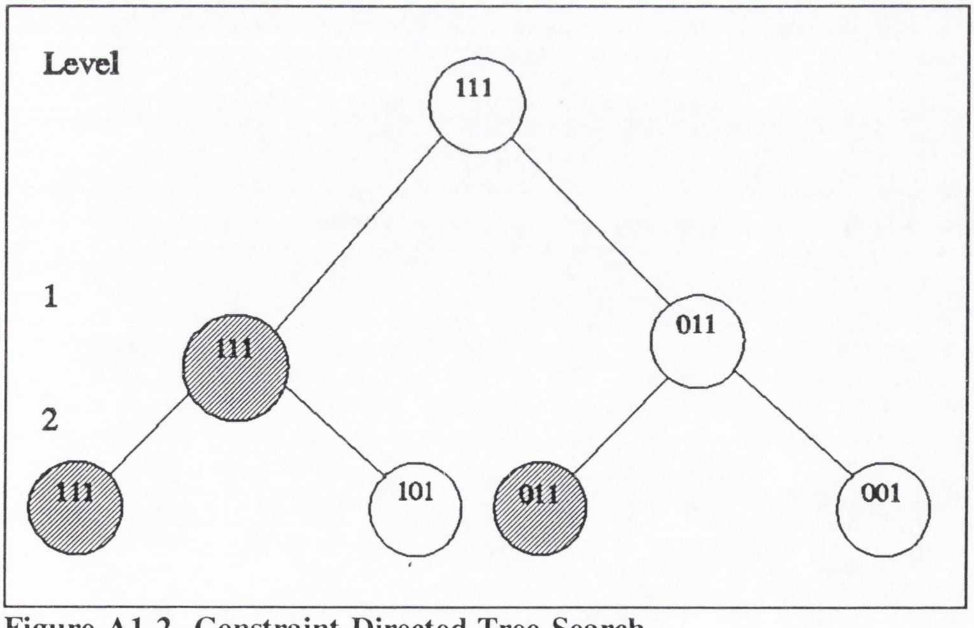

Figure A1.2 Constraint-Directed Tree Search

The removal of a unit is considered at each level of the search tree. If there is sufficient generating capacity at a node, all descendants of the node may be removed from the search, since these descendants will have progressively less capacity. This condition may be summarised simply as:

$$\sum_{i \in C} \phi_i \, u_i < D \tag{A1.4}$$

where C is the set of committed units.

The cost of feasible combinations are obtained using an economic dispatch algorithm, and the cheapest retained as the current optimum. The dispatch algorithm is therefore an integral part of the commitment process for each interval. This algorithm is now outlined.

A1.2 Economic Dispatch Formulation

The economic dispatch problem may be formulated as a linear program, and solved using the dual simplex method. For a given unit commitment, the no-load costs are fixed, and it is required to minimise the remaining generating cost over the dispatch interval. The cost for interval j may be summarised as:

$$Z = \sum_{i \in C} b_i \, x_i \, T_j \tag{A1.5}$$

Each generating unit operates between upper and lower limits:

$$l_i \le x_i \le u_i \qquad \textit{for } i \in C \tag{A1.6}$$

The total generated output must equal the demand. To conform with the requirements of the linear program, the following inequality constraint is used:

$$\sum_{i \in C} x_i \geq D \tag{A1.7}$$

If a solution exists, equation A1.7 will be satisfied as an equality, since any extra generation would increase the cost needlessly.

A further constraint on the solution is that each infeed, or a proportion of it, must be covered by the total emergency reserve from the remaining units. The reserve characteristics for a unit are described by the following equations and by Figure A1.3:

$$\left. \begin{array}{l} r_i \leq C1_i + S1_i \; x_i \\ r_i \leq C2_i + S2_i \; x_i \\ r_i \leq C3_i + S3_i \; x_i \end{array} \right\} \quad for \; i \in C \tag{A1.8}$$

$C1_i$, $C2_i$, $C3_i$, $S1_i$, $S2_i$, and $S3_i$ are constants associated with unit i. The values of these constants depend on unit dynamics.

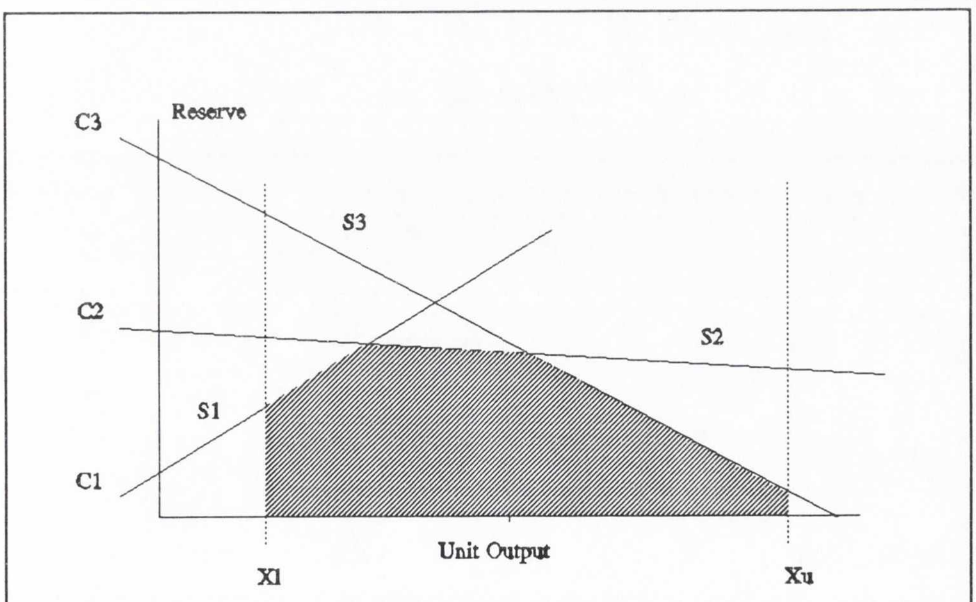

Figure A1.3 Unit Emergency Reserve Characteristics

The following constraint equations ensure that each infeed is covered by sufficient emergency reserve:

$$x_j \leq \sum_{i \in C} r_i - r_j \qquad j \in C \tag{A1.9}$$

The emergency reserve variables r_i may be removed from the main minimisation problem. The primary problem involving variables x_i is then solved. The x variables are then made available to a reserve sub-problem which determines, for each unit in turn, which of equations A1.8 is the most restrictive. The corresponding inequality constraint then becomes an equality relating the emergency reserve from each unit to the unit output. The most violated reserve constraint is then adjoined to the main problem as an inequality:

$$-x_j + \sum_{i \in C} S_i \, x_i - S_j \, x_j \geq -\sum_{i \in C} C_i + C_j \qquad j \in C \tag{A1.10}$$

The main problem, including equation A1.10, is solved, and the provisional solution presented once more to the reserve sub-problem. The process is repeated until equation A1.9 is satisfied for all j, in which case an overall solution has been found. Note that in small isolated power systems it may not be economic to cover all of the largest infeed lost. This can be accounted for by substituting $-Rx_j$ for $-x_j$ in equation A1.10, with $0 \leq R \leq 1$. Following the loss of unit j, $(1-R)x_j$ MW of load should be disconnected by load shedding relays.

A1.3 Dispatch Problem Solution

The economic dispatch problem may be summarised as follows:

Minimise

$$Z = x^T b \tag{A1.11}$$

subject to

$$x^T A \geq l^T \tag{A1.12}$$

x^T is a $1 \times n$ matrix comprising the n variable unit outputs; b is the incremental cost vector; A is an $n \times N$ matrix of constraint coefficients, corresponding to the N constraints on x; and l^T is a $1 \times N$ matrix, corresponding to the constraint limits.

The solution may be obtained using linear programming methods. The following procedure is equivalent to the dual simplex method. The process is started by taking n inequalities as equalities to give a basic solution x_b^T for n unknowns. Thus,

$$x_b^T B = l_b^T \tag{A1.13}$$

where the $n \times n$ matrix B, consisting of n columns of A, is the basic matrix. x_b^T is chosen to be the cheapest solution; in the dispatch problem this will obviously be when all units are at their lower limits. This is known as an 'optimistic' basic solution because it may violate some of the remaining N-n constraints. B will, in this case, be the $n \times n$ identity matrix, so that the initial basic solution is easily found:

$$x_b^T = l_b^T B^{-1} \tag{A1.14}$$

If no non-basic constraints are violated, the optimistic solution will also be the optimum solution. Suppose the non-basic constraint j is violated, then,

$$x_b^T \; a_j < l_j \tag{A1.15}$$

where a_j is the jth column of A. Let x_b become $x=xb+\Delta x$ to satisfy the jth constraint. The basic constraints must still be satisfied. Therefore,

$$(x_b+\Delta x)^T \; B \geq l_b^T \tag{A1.16}$$

Therefore, using equation A1.13,

$$\Delta x^T \; B \geq 0 \tag{A1.17}$$

It is required that,

$$(x_b+\Delta x)^T \; a_j \geq l_j \tag{A1.18}$$

Therefore,

$$\Delta x^T \; a_j \geq l_j - x_b^T \; a_j \tag{A1.19}$$

Therefore,

$$(\Delta x^T \; B)(B^{-1} a_j) \geq l_j - x_b^T a_j \tag{A1.20}$$

The right-hand side is the constraint violation, and is positive - refer to equation A1.15. Letting

$$\Delta l_b^T = \Delta x^T \; B \quad also \quad y_j = B^{-1} \; a_j \tag{A1.21}$$

we have

$$\Delta l_{b1}y_{1j}+\ldots+\Delta l_{bk}y_{kj}+\ldots+\Delta l_{bn}y_{nj} \geq l_j - x_b^T \; a_j \tag{A1.22}$$

Δl_{bk} corresponds to a relaxation of basic constraint k away from the limit by $\Delta x^T a_k$.

It is necessary to reduce the constraint j violation (l_j - x_b^T a_j) to zero. Since Δx^T $B \geq 0$, $\Delta l_b^T \geq 0$, a positive element y_{kj} must be found, and Δl_{bk} relaxed, such that,

$$\Delta l_{bk}y_{kj} = l_j - x_b^T \; a_j \tag{A1.23}$$

If there is no positive y_{kj}, no solution exists. If a positive y_{kj} is found, then,

$$(x_b+\Delta x)^T \; a_j = l_j \tag{A1.24}$$

The jth constraint is now satisfied as an equality and becomes a member of the new basis, replacing the relaxed kth constraint. The new basic solution is,

$$\hat{x}_b^T = \hat{l}_b^T \; \hat{B}^{-1} \tag{A1.25}$$

where $\hat{l}_b$ is l_b with l_{bk} replaced by l_j and $\hat{B}$ is B with the kth column replaced by a_j. Writing

$$\begin{aligned} \hat{B} &= B[e_1 \ldots e_{k-1} B^{-1} a_j e_{k+1} \ldots e_n] \\ &= BE_{kj} \end{aligned} \tag{A1.26}$$

where E_{kj} is the identity matrix with the kth column replaced by $y_j = B^{-1}a_j$, the inverse of the new basic matrix may be readily evaluated from

$$\hat{B}^{-1} = E_{kj}^{-1} \; B^{-1} \tag{A1.27}$$

The choice of the constraint to be relaxed, k, is such as to give the minimum cost to overload reduction ratio. The extra cost is,

$$\Delta Z = \Delta x^T b \tag{A1.28}$$

Therefore,

$$\Delta Z = (\Delta x^T B)(B^{-1} b) \tag{A1.29}$$

Therefore,

$$\Delta Z = \Delta l_b^T w_b \tag{A1.30}$$

where $w_b = B^{-1}b$. Relaxing constraint k by Δl_{bk}, the overload reduction is $\Delta l_{bk} y_{kj}$ at an extra cost of $\Delta Z = \Delta l_{bk} w_{bk}$. Thus the constraint k to be removed from the basis is that for which w_{bk}/y_{kj}, with $y_{kj} > 0$, is a minimum. It can be shown that any departure from this rule invalidates the optimisation process.

The solution therefore moves from one optimistic solution to another (more expensive) one. An optimum solution, in which all non-basic constraints are satisfied, will be reached in a finite number of steps, provided a feasible solution exists. It should be noted that the violated constraint j to be introduced into the basis need not be the most overloaded.

A1.4 NIE System and Reserve Data

The current NIE unit ratings, costs and emergency reserve data are outlined in Tables A1.1 and A1.2. This data was used for all economic loading calculations performed in the thesis.

xl MW	xu MW	a £/h	b_1 £/MWh	ECR MW	b_2 £/MWh	SC £	τ h	RTIME h	λ h^{-1}
50.0	250.	206.46	8.38	500	8.38	1225.5	10	1	0.00010
50.0	250.	206.46	8.38	500	8.38	1225.5	10	1	0.00010
50.0	180.	160.00	7.89	500	7.89	1225.5	10	1	0.00010
50.0	180.	160.00	7.89	500	7.89	1225.5	10	1	0.00010
50.0	120.	96.71	9.52	500	9.52	958.9	10	1	0.00010
50.0	120.	96.71	9.52	500	9.52	958.9	10	1	0.00010
50.0	120.	96.71	9.52	500	9.52	958.9	10	1	0.00010
50.0	200.	140.01	9.36	500	9.36	1457.7	10	1	0.00010
50.0	200.	140.01	9.36	500	9.36	1457.7	10	1	0.00010
50.0	200.	140.01	9.36	500	9.36	1457.7	10	1	0.00010
15.0	60.0	00.00	29.16	500	29.16	0	10	1	0.00010
18.0	60.0	00.00	29.16	500	29.16	0	10	1	0.00010
15.0	60.0	69.06	12.30	48	14.26	643.2	10	1	0.00010
15.0	60.0	75.44	12.29	48	14.99	643.2	10	1	0.00010
15.0	60.0	75.41	12.26	48	15.17	643.2	10	1	0.00010
15.0	60.0	75.01	12.28	48	15.19	643.2	10	1	0.00010
15.0	60.0	77.29	12.31	48	15.23	643.2	10	1	0.00010
15.0	30.0	52.21	17.55	24	20.34	738.61	10	1	0.00010
15.0	30.0	53.56	17.50	24	20.36	738.61	10	1	0.00010
15.0	60.0	94.82	16.13	48	19.67	1305.27	10	1	0.00010
15.0	60.0	86.61	16.39	48	20.06	1305.27	10	1	0.00010
15.0	60.0	85.94	16.50	48	20.22	1305.27	10	1	0.00010
15.0	60.0	00.00	29.16	500	29.16	0	10	1	0.00010
15.0	30.0	00.00	29.16	500	29.16	0	10	1	0.00010
18.0	60.0	00.00	29.16	500	29.16	0	10	1	0.00010
18.0	60.0	00.00	29.16	500	29.16	0	10	1	0.00010
18.0	30.0	00.00	29.16	500	29.16	0	10	1	0.00010

Table A1.1 NIE System Data

xl = lower limit of unit
xu = upper limit of unit
a = no-load cost of unit
b_1 = first incremental cost of unit
ECR = economic continuous rating
b_2 = second incremental cost of unit
SC = cold start-up cost of unit
τ = cooling time constant of unit
RTIME = run-up time of unit
λ = failure rate of unit

(In this table , ECR > xu implies a unit with a single incremental cost, i.e., $b_1 = b_2$)

C1 MW	C2 MW	C3 MW	S1	S2	S3	Inst	time
53.40	500.0	201.80	-0.046	0.0	-0.807	1	1000
53.40	500.0	201.80	-0.046	0.0	-0.807	1	1000
16.00	500.0	126.30	0.002	0.0	-0.702	1	1000
16.00	500.0	126.30	0.002	0.0	-0.702	1	1000
15.90	500.0	98.20	-0.015	0.0	-0.818	1	1000
15.30	500.0	86.8	-0.014	0.0	-0.724	1	1000
12.30	500.0	87.6	-0.011	0.0	-0.730	1	1000
21.20	500.0	126.0	-0.013	0.0	-0.630	1	1000
40.50	500.0	168.4	-0.082	0.0	-0.842	1	1000
40.70	500.0	168.8	-0.033	0.0	-0.844	1	1000
7.70	500.0	28.1	-0.029	0.0	-0.937	1	1000
7.70	500.0	28.1	-0.029	0.0	-0.937	1	1000
15.20	500.0	56.30	-0.020	0.0	-0.939	1	1000
15.20	500.0	56.30	-0.020	0.0	-0.939	1	1000
15.20	500.0	56.30	-0.020	0.0	-0.939	1	1000
15.20	500.0	56.30	-0.020	0.0	-0.939	1	1000
15.20	500.0	56.30	-0.020	0.0	-0.939	1	1000
6.40	500.0	27.8	-0.022	0.0	-0.926	1	1000
6.40	500.0	27.8	-0.022	0.0	-0.926	1	1000
12.70	500.0	55.70	-0.020	0.0	-0.928	1	1000
12.70	500.0	55.70	-0.020	0.0	-0.928	1	1000
12.70	500.0	55.70	-0.020	0.0	-0.928	1	1000
12.70	500.0	55.70	-0.020	0.0	-0.928	1	1000
12.70	500.0	55.70	-0.020	0.0	-0.928	1	1000
12.70	500.0	55.70	-0.020	0.0	-0.928	1	1000
12.70	500.0	55.70	-0.020	0.0	-0.928	1	1000
12.70	500.0	55.70	-0.020	0.0	-0.928	1	1000

Table A1.2 NIE Reserve Data

C1, C2, C3, S1, S2, S3 - refer to Fig A1.3

Inst = initial state of unit, 'o' off, '1' on
time = the length of the initial state time for unit (h)
(only relevant if unit is off, '1000' is a dummy value)

A2.1 Initial Estimates for Autogressive Models

The general AR(p) model is represented as :

$$y(t)=\phi_1 y(t-1)+\phi_2 y(t-2)+....+\phi_p y(t-p)+e(t) \tag{A2.1}$$

If both sides of equation A2.1 are multiplied by y(t-k), where k = 1, 2, 3, ...,p, then :

$$\begin{aligned} y(t-k)y(t)=\phi_1 y(t-k)y(t-1)+\phi_2 y(t-k)y(t-2)+....+ \\ \phi_p y(t-k)y(t-p)+y(t-k)e(t) \end{aligned} \tag{A2.2}$$

Then taking the expected value of both sides of equation A2.2 and assuming stationarity yields :

$$\gamma(k)=\phi_1\gamma(k-1)+\phi_2\gamma(k-2)+....+\phi_p\gamma(k-p) \tag{A2.3}$$

However, as shown in Chapter Two,

$$\rho(k)=\frac{\gamma(k)}{\gamma(0)} \tag{2.3.9}$$

If k = 1, 2, 3, ..., p then the following system of equations, known as the Yule-Walker equations, are obtained :

$$\begin{aligned}
\rho(1) &= \phi_1 \qquad\quad\;\; +\phi_2\rho(1)+\ldots. \quad +\phi_p\rho(p-1) \\
\rho(2) &= \phi_1\rho(1) \quad +\phi_2 \qquad\quad +\ldots. \quad +\phi_p\rho(p-2) \\
&\;\;\vdots \\
\rho(p) &= \phi_1\rho(p-1)+\phi_2\rho(p-2)+\ldots. \;\; +\phi_p
\end{aligned} \tag{A2.4}$$

Since the theoretical autocorrelations ρ(p) are unknown, the estimates r(p) are used instead. The Yule-Walker equations can then be solved for ϕ_1, ϕ_2, ..., ϕ_p to obtain the initial estimates.

A2.2 Initial Estimates for Moving Average Models

The general MA(q) model is represented as :

$$y(t)=e(t)-\theta_1 e(t-1)-\theta_2 e(t-2)-\ldots.-\theta_q e(t-q) \tag{A2.5}$$

Multiplying both sides by y(t-k), where k = 1, 2, 3, ..., q, yields :

$$\begin{aligned}
y(t-k)y(t) &= [e(t)-\theta_1 e(t-1)-\theta_2 e(t-2)-\ldots.-\theta_q e(t-q)] \\
&\quad \times[e(t-k)-\theta_1 e(t-k-1)-\theta_2 e(t-k-2)-\ldots.-\theta_q e(t-k-q)]
\end{aligned} \tag{A2.6}$$

Taking the expected values on both sides of the equation (A2.6) gives

$$\begin{aligned}
\gamma(k) &= E[(e(t)-\theta_1 e(t-1)-\theta_2 e(t-2)-\ldots.-\theta_q e(t-q)) \\
&\quad \times(e(t-k)-\theta_1 e(t-k-1)-\theta_2 e(t-k-2)-\ldots.-\theta_q e(t-k-q))]
\end{aligned} \tag{A2.7}$$

$$\begin{aligned}
\gamma(k) = E(&e(t)e(t-k)-\theta_1 e(t)e(t-k-1)-\ldots.-\theta_q e(t)e(t-q-k) \\
&-\theta_1 e(t-1)e(t-k)+\theta_1^2 e(t-1)e(t-k-1)+\ldots.+\theta_1\theta_q e(t-1)e(t-k-q) \\
&-\theta_2 e(t-2)e(t-k)+\theta_2\theta_1 e(t-2)e(t-k-1)+\ldots.+\theta_2\theta_q e(t-2)e(t-k-q) \\
&\;\cdot\cdot \\
&\;\cdot\cdot \\
&-\theta_q e(t-q)e(t-k)+\theta_q\theta_1 e(t-q)e(t-k-1)+\ldots.+\theta_q^2 e(t-q)e(t-k-q))
\end{aligned} \tag{A2.8}$$

The expected value of equation (A2.8) will depend upon the value of k. In general for k=0, equation (A2.8) becomes :

$$\gamma(k) = -\theta_k\sigma_e^2 + \theta_1\theta_{k+1}\sigma_e^2 + \theta_2\theta_{k+2}\sigma_e^2 + ... + \theta_{q-k}\theta_q\sigma_e^2$$

or

$$\gamma(k) = (-\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2} + ... + \theta_{q-k}\theta_q)\sigma_e^2 \tag{A2.9}$$

If $\rho(k) = \gamma(k)/\gamma(0)$, and

$$\gamma(0) = (1 + \theta_1^2 + \theta_2^2 + ... + \theta_q^2)\sigma_e^2$$

Then

$$\rho(k) = \frac{(-\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2} + ... + \theta_{q-k}\theta_q)\sigma_e^2}{(1 + \theta_1^2 + \theta_2^2 + ... + \theta_q^2)\sigma_e^2} \tag{A2.10}$$